A Gaussian Mixture Method for Specific Differential Phase Retrieval at X-band Frequency

Guang Wen[1], Neil I. Fox[1,*], and Patrick S. Market[1]
[1]School of Natural Resources, University of Missouri, 332 ABNR Building, Columbia, Missouri, USA, 65201
Correspondence: Neil I. Fox (foxn@missouri.edu)

We would like to express our sincere thanks to the Editor for the efficient review management, and to the anonymous reviewers for their valuable comments and suggestions. We have addressed all the reviewers' comments point by point in the revision.

Review #2

1 *General comments*

The method is quite complicated and as such it would be amazing if the authors considered implementing it in a radar toolkit such as for example Py-ART https://arm-doe.github.io/pyart/, which already contains a few retrieval methods.

[response] Indeed, the Gaussian mixture method is relatively complicated when compared to the linear regression method, since it provides the variance of K_{dp} together with the mean K_{dp}. It is not easy for all the readers to implement it via Anaconda Python. Yes, we are very glad that the GMM is included in the Py-ART software package. We intend to make the codes available to the community soon. As we described in section 3 (Data), we developed the algorithm using Anaconda Python on RedHat Linux 7, and used the Py-ART to get the inputs and outputs and draw our figures. We think it should be straightforward to implement it in the Py-ART. Thus, the users can derive the best mean K_{dp} for QPE using the methods such as Vulpiani et al. (2012) and Giangrande et al. (2013), and obtain the variance of K_{dp} for studying the propagation of uncertainty in the weather or hydrological model using the Gaussian mixture method.

2 *Major comments*

2.1. ***Section 4.3 and later : I think that you should use a different notation for the raw measured differential phase shift on propagation and the filtered version with δ_{co} removed. Usually the notation Ψ_{dp} is used for the raw measurement and Φ_{dp} for the filtered signal from which K_{dp} is estimated***

[responses] Our notations were very ambiguous in the first manuscript. We have fixed this problem by denoting the data as Ψ_{dp} before the elimination of the backscattering differential phase shift, and as Φ_{dp} after it.

[changes] We have made a number of changes through the paper, for example:

p.7, ln.20–30: From the chart of LR in Fig. 1.a, we can see that after the radar measurements are collected, the Ψ_{dp} is unfolded, and then the clutter is removed. After these corrections, an iterative filtering method is applied to the Ψ_{dp} profile. An adaptive method is finally used to estimate the K_{dp} profile according to the values of Z_H. The Gaussian mixture model, on the other hand, processes Ψ_{dp} differently. First of all, the clutter is masked out according to the thresholds of Z_H and the variation of Ψ_{dp}. Secondly, the range r and Ψ_{dp} are fitted into a Gaussian mixture to yield the joint PDF, while the Ψ_{dp} mean and the Ψ_{dp} variance are obtained by taking the first raw and second central moments of the conditional PDF of Ψ_{dp} given r. Thirdly, some specific clusters in the Gaussian mixture PDF are adjusted to solve the problems of ambiguous Ψ_{dp} and backscattering differential phase shift δ_{co} in order to derive the PDF of Φ_{dp}. Fourthly, a raw K_{dp} profile is calculated from the first derivative of the expected values of Φ_{dp}, and the associated variances are obtained via a Taylor series expansion. Finally, the raw K_{dp} profile is smoothed, and consequently, the variances are reduced. In addition, new Φ_{dp} with lower variances can be re-constructed from the K_{dp} estimates.

2.2. ***One major issue in an operational context is the computational cost of these more sophisticated K_{dp} retrieval techniques. Mainly for this reason, the standard linear regression methods are still the norm. Could you discuss and provide numbers for the computational cost of your method and maybe compare it with other methods?***

[responses] It is true that the GMM takes more computation time than the LR. For the PPI data used in section 5, the LR takes about 1.47 seconds for phase unfolding on the PC, 0.458 seconds for Ψ_{dp} smoothing and 0.109 seconds for regression-based K_{dp} estimation. In total, the LR takes about 2.037 seconds if we ignore the time used for the inputs and outputs. On the other hand, the GMM uses 2.99 seconds for data masking, 2.348 seconds for Ψ_{dp} density estimation, 0.73 seconds for Ψ_{dp} unfolding and δ_{co} elimination and 0.98 seconds for K_{dp} estimation. In total, we need 7.058 seconds for this case. If we skip the data masking process, we need about 4.068 seconds, about twice than the LR. Nevertheless, the GMM can obtain more information from the radar data than the LR.

[changes] p.16, ln. 26–29: Moreover, the computational time is crucial for the real-time application of the K_{dp} retrieval algorithms. For the data in Fig. 8, the GMM takes about 7.058/4.068 seconds to process the K_{dp} with/without the data masking, whereas the LR reduces the time to about 2.037 seconds. It indicates that the LR has the advantages of simplicity and efficiency. Nevertheless, the GMM can obtain more information from the radar data, which is useful for the model studies.

2.3. ***In the conclusion, I think it would be interesting to discuss if this method could be used as such for other frequencies (C-band and S-band in particular) or if it would require some relevant tweaks.***

[responses] We think the X-band is the most difficult among the three wavelengths, since it leads to the most significant phase changes and backscattering differential phase shift (δ_{co}). The Gaussian mixture method can definitely be adapted to C-band and S-band radars with a few modifications. For example, we need to adjust the thresholds for the textures used in the quality control. For CP2 radar (S-band), we need $\sigma(\Phi_{dp}) < 7.5°$ for weather echoes, while the threshold increases to 10° for the CPOL radar (C-band), combined with the ρ_{hv} threshold. In a similar manner, we also need to adjust the thresholds for Ψ_{dp} folding and δ_{co} elimination. Nevertheless, the steps for the calculations of the joint PDF of Ψ_{dp} and K_{dp} will be unchanged.

[changes] We have made a number of changes:

p. 19, ln. 3–6: In the future study, the algorithm will also be extended to other frequencies, such as C-band (Vulpiani et al., 2012; May et al., 1999) and S-band (Bringi and Chandrasekar, 2001). The thresholds in the data masking, the Ψ_{dp} unfolding and the δ_{co} elimination will be adjusted according to the radar specifications. Nevertheless, the steps for the calculations of the PDFs of Ψ_{dp} and K_{dp} will be remained.

3 *Minor comments*

3.1. ***p2. l.11-12 : This sentence is not very clear and syntactically correct, please reformulate***

[changes] It has been changed to

p.2, ln. 13–14: In addition, Gorgucci et al. (1999) note that the nonuniform rainfall path produces large errors in the K_{dp} estimates, while the errors increase as the radar reflectivity varies in dimensions.

3.2. ***p2 : l.23 : Like the proposed method, the Kalman filter method also provides an estimate of the standard deviation of the retrieved KDP at X-band, it would be interesting to explain it in in broader detail as well as discuss the differences and respective advantages of both methods.***

[responses] The Kalman filter method is also an excellent method for the K_{dp} estimation, since it can simultaneously obtain the K_{dp}, the attenuation-corrected Z_H, the attenuation-corrected Z_{DR} and δ_{co}. The method is then adapted to various environmental conditions by considering Ψ_{dp} only. The Kalman filter method significantly increases the accuracy of the K_{dp} mean when compared to the linear regression method, and gives some improvements when estimating K_{dp} in the small-scale storm structure. However, it is very difficult to compare the GMM to the Kalman filter without the original codes. From Schneebeli et al. (2014), we can see that the primary difference is that the Kalman filter method assumes the error covariance function follows a linear Gaussian distribution, whereas the GMM considers the joint PDF of r and Ψ_{dp} as a non-linear Gaussian mixture. Thus, the GMM may have better performance when the data are multimodal. Moreover, the Kalman filter method derive a priori from the measured DSD, leading to some constraints on the particle types. In contrast, the GMM fits the data to obtain the random errors of Ψ_{dp} data.

[changes] We have made a number of changes:

p.2, ln. 26–27: It is noticeable that the Kalman filter method minimizes the Gaussian error function to obtain the mean profile of K_{dp}. It gives a significant improvement on the K_{dp} mean, particularly in the small-scale structure with high peaks.

p.2, ln. 35–p.3 ln.1: When compared to the existing methods, our method considers the joint probability density function of the data as the non-linear Gaussian mixture, leading to better performance for the multimodal data.

3.3. ***p11: l.3-5: I have trouble understanding this paragraph. I would suggest to reformulate to make it clearer, in particular the term "transformed into the next stage" is inappropriate.***

[changes] This paragraph has been changed to

p. 11, ln. 13–17: On the other hand, the Ψ_{dp} unfolding is more straightforward in GMM. Figure 5 shows the flowchart of the Ψ_{dp} unfolding and the δ_{co} elimination. After obtaining the PDF of Ψ_{dp}, the initial step of the Ψ_{dp} unfolding selects the density ellipses with at least 6 data points. Next, the second step calculates the difference of the means μ_i between the two consecutive density ellipses along the range. At this point, the PDF of Ψ_{dp} is ready to be corrected for ambiguous Ψ_{dp}. In the final step, the mean of the latter density ellipse is finally added up 180°, if the former mean is larger than the latter one by 80°.

3.4. ***I would suggest to add another flowchart for the step ϕ_{dp} unfolding and δ_{co} estimation.***

[changes] We have added a new flowchart in Fig. 5 for the steps of the Ψ_{dp} unfolding and the δ_{co} elimination.

3.5. ***p.13 l.17-18: It would be good to discuss why you choose this particular FIR filter. I am also not sure how the number of considered gates is defined.***

[responses] In this paper, we create a base form of the FIR filter using a window method. The cut-off frequency (or Nyquist frequency) is 0.053 and the window is generated by a Gaussian distribution with standard deviation of 28. These coefficients yield the best performance for the MZZU radar when the iterative filtering is applied (Hubbert and Bringi, 1995). We then tune the order of the FIR filter (gate number). For example, we select the gate number of 31 if the relative square error between the gate number of 31 and the gate number of 33 is below 0.001. The gate number frequently falls between 29 and 33 for the MZZU radar.

[changes] We have made a number of changes:

p.13, ln. 31–32: Figure 7 shows the time responses of the FIR with the cutoff frequency of 0.053 and the Gaussian window of 28, which yield the best performance for the MZZU radar.

p.14, ln. 1–4: In this study, we gradually increase the order number to calculate the difference between the K_{dp} profiles obtained by the FIR filters with two adjacent order numbers. The optimal order of the FIR filter is then set when the relative square error of the two K_{dp} is below 0.001. For profiles with sufficiently large data points, the order number is between 29 and 33 for the MZZU radar.

3.6. ***p.16 l.8: It would be good to include one or two sentences that explain briefly this X-band rainfall rate algorithm.***

[changes] Yes, we have given the rain rate algorithm in this revision.

p. 15, ln. 6–9: The radar hourly rain amount is calculated based on the CASA radar rainfall algorithm, which is given as (Wang and Chandrasekar, 2010; Chen and Chandrasekar, 2015)

$$R(K_{dp}) = 18.15 K_{dp}^{0.79}, \quad (1)$$

when R is the instantaneous rain rate in mm h^{-1}.

3.7. ***Figures 4 and 5 should be visually improved. In particular the data points are too hard to see because of the error bars. I would for example replace the error bars by thin lines located one each side of the plot. Also the limits of the y axis could be adjusted.***

[changes] We have updated Figs. 4 and 6 according to this comment.

3.8. ***Figure 9: It would be useful to also include the radar estimates derived from the LR Kdp.***

[changes] We have included the rain estimates derived from LR K_{dp} in Fig. 10 in this revision.

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