Response to Reviewer 2 comments

We thank the reviewer for their detailed comments and feel the changes we have made in response greatly improve the manuscript. Our responses to the individual points are detailed below.

2.1. “It seems that the authors are, in fact, just using one method…”

We take the reviewer’s point, and we would go even further and say that both methods reflect the assumption that the concentration distribution is assumed to be of a Gaussian form. However, we would argue that the two methods of solution are so different that they warrant separate sections. In addition, the approach to the solution when there is no significant temperature inversion present (“Method 1”) is novel, and alone justifies the separation into two “Methods”. Taking into account the above, we have changed the text (extra text in red):

“Two different analysis approaches have been used, determined by the outcome of these measurements. They are referred to as Solution Method 1 and Solution Method 2 in this manuscript. Both solution methods reflect the assumption that the concentration distribution is assumed to be of a Gaussian form. However, the techniques of solution are different, and are here split into separate sections.”

We have now changed the subsection headings to “Solution Method 1” and “Solution Method 2”. We hope this now creates the distinction between the two solutions, while reflecting the fact that the solutions themselves refer to a single underlying assumption – the Gaussian assumption.

2.2 Presentation of the methods

We apologise for the poor presentation of the method and the equations. Hopefully the revised manuscript is an improvement. Specific changes to the manuscript bearing upon this point are described below.

2.2.1. “Eq. 2 is clearly incorrect…”

The reviewer is correct, this was a mistake in our original manuscript submission. The large round bracket at the end of our original eqn. (2) should have been used to terminate the first exponential term: this then corresponds to the reviewer’s derivation and subsequent equation.

Our original equation (2):

\[
C(x, y, z) = \frac{q}{2\pi \sigma y \sigma z U} \exp \left( -\frac{(y - y_0)^2}{2\sigma_y^2} \right) \left[ \exp \left( \frac{z^2}{2\sigma_z^2} \right) + \exp \left( -\frac{(z + 2H)^2}{2\sigma_z^2} \right) \right] + \exp \left( -\frac{(z - 2H)^2}{2\sigma_z^2} \right) \]

Should have been:
\[ C(x,y,z) = \frac{q}{2\pi \sigma_y \sigma_z U} \exp \left( -\frac{(y-y_0)^2}{2\sigma_y^2} \right) \left[ \exp \left( \frac{z^2}{2\sigma_z^2} \right) + \exp \left( -\frac{(z+2H)^2}{2\sigma_z^2} \right) \right] \]

This has been corrected in the text. We are grateful to the reviewer for pointing this out.

2.2.2 Nomenclature and the x, y and z dependence

“\( \sigma_y \) and \( \sigma_z \) are both functions x and I’m used to computing these based on the atmospheric stability class… it wasn’t clear to me, until much later – that the authors were recomputing the dispersion parameters based on each transect (that is what you are doing, right?)”

The reviewer is correct here. The dispersion parameters \( \sigma_y \) and \( \sigma_z \) are estimated from the measurements. Although attempts have been made (e.g. Song et al. 2003: reference below) to adopt Briggs-type formulas for use over sea surfaces, classified according to atmospheric stability, such ad-hoc approaches depend upon a simple manipulation of the land-based formulas (in the Song et al. example, a simple multiplication factor is used). This is conceivably appropriate for the lateral dispersion of gas but the vertical dispersion coefficient is unlikely to be represented well by this type of approach. The land-based formulas are derived from a large body of experiments: this is not the case for the marine equivalent (indeed, this would be a good subject for further study). Thus, we calculated the lateral and (when appropriate) vertical dispersion parameters based upon the aircraft measurements. This was perhaps not clear in the original manuscript, so we have added the extra text

“In land-based dispersion modelling, it is common to employ an approximation to the dispersion parameters \( \sigma_y \) and \( \sigma_z \). (Examples may be found in Turner 1994.) These approximations (derived from many field experiments) are based upon the atmospheric stability and distance from source. Some attempts (e.g. Song et al. 2003) have been made to find similar approximations over sea surfaces; such attempts are not the result of field experiment, but rather of a manipulation of land-based formulae, and there is a question as to their validity. Thus, in the present study, we derive the dispersion parameters from the aircraft measurements, as described below.”

This appears early in the manuscript to hopefully prevent the lack of clarity mentioned by the reviewer.

We have added the Song et al. reference to the manuscript.


“Related to this point, Eq. 4-7 are confusing because… is this the same \( C_0 \) in all these cases?..”
We have re-written Eq. 4-7 to show the full x, y, z dependence. Also we have added the extra text in the Solution Method 2 (formerly “Method 2”) subsection: “N. B. the $C_0$ here is different to the $C_0$ for Solution Method 1.” Hopefully this will alert readers to the different forms of the $C_0$ term.

“Additionally…undefined $U’$…undefined $H’$…”

Apologies. These are typos and should be $U$ and $H$ respectively – now corrected.

3.

Minor comments:

3.1 “What about the reflection at the ground…”

We agree that reflections at the surface do need to be accounted for. Below is the relevant, full equation taken directly from Turner, *Workbook of Atmospheric Dispersion Estimates* (CRC Press, 1994; referenced in the text):
Chapter 2.

Figure 2.2 Coordinate system showing Gaussian distributions in the horizontal and vertical.

The variables used are:

$\chi$  \hspace{1cm} Air pollutant concentration in mass per volume, usually $g \text{ m}^{-3}$.

$Q$  \hspace{1cm} Pollutant emission rate in mass per time, usually $g \text{ s}^{-1}$.

$u$  \hspace{1cm} Wind speed at the point of release, $m \text{ s}^{-1}$.

$\sigma_y$  \hspace{1cm} The standard deviation of the concentration distribution in the crosswind direction, $m$, at the downwind distance $x$.

$\sigma_z$  \hspace{1cm} The standard deviation of the concentration distribution in the vertical direction, $m$, at the downwind distance $x$.

$\pi$  \hspace{1cm} The mathematical constant $\pi$ equal to 3.1415926...

$H$  \hspace{1cm} The effective height of the centerline of the pollutant plume.
The notation used following $\chi$ in parentheses is to give the three coordinates of the receptor location according to the coordinate scheme described above. Following a semicolon, the effective height of emission of the source is given.

The equation is given as four separate factors which are multiplied times each other. These four factors represent the dependency upon emissions, or the source factor, and what occurs in the three dimensions parallel to the three coordinate axes.

$$
\chi(x,y,z;H) =
$$

Emissions factor 

$$Q$$

Downwind factor 

$$\frac{1}{u}$$

Crosswind factor 

$$\frac{1}{(2\pi)^{1/2}\sigma_y} \exp \left[ -\frac{y^2}{2\sigma_y^2} \right]$$

Vertical factor 

$$\frac{1}{(2\pi)^{1/2}\sigma_z} \left\{ \exp \left[ -\frac{(H-z)^2}{2\sigma_z^2} \right] + \exp \left[ -\frac{(H+z)^2}{2\sigma_z^2} \right] \right\}$$

(2.1)

A brief explanation of the four terms follows.

1. The concentrations at the receptor are directly proportional to the emissions.

2. Parallel to the x axis, the concentrations are inversely proportional to wind speed as explained in Chapter 1.

3. Parallel to the y axis, that is, crosswind, the concentrations are inversely proportional to the crosswind spreading, $\sigma_y$, of the plume; the greater the downwind distance from the source, the greater the horizontal spreading, $\sigma_y$, the lower the concentration. The exponential involving the ratio of $y$ to $\sigma_y$ just corrects for how far off the center of the distribution the receptor is in terms of standard deviations. The receptor is $y$ from the center since the crosswind distribution center is at $y = 0$, that is, directly above the x-axis.

4. Parallel to the z axis, that is, vertical, the concentrations are inversely proportional to the vertical spreading of the plume, $\sigma_z$; the greater the downwind distance from the source, the greater the vertical dispersion and the lower the concentration. The sum of the two exponential terms in the vertical factor represent how far the receptor height, $z$, is
In Turner’s notation, \( \chi \) is the concentration, \( \sigma_y \) and \( \sigma_z \) are the dispersion parameters (we adopted the same notation), \( u \) is the ambient wind speed, \( H \) is the height of the source above the surface, and \( z \) is the height of the receptor. With a source at the surface (\( H = 0 \)), Turner’s (2.1) gives

\[
\chi(x, y, z; 0) = \frac{Q}{2 \pi u \sigma_y \sigma_z} \exp \left( -\frac{y^2}{2 \sigma_y^2} \right) \exp \left( -\frac{z^2}{2 \sigma_z^2} \right) + \exp \left( -\frac{(H + z)^2}{2 \sigma_z^2} \right)
\]

Adopting our notation (\( C \equiv \chi ; U \equiv u ; q \equiv Q \)) then we obtain

\[
C(x, y, z; 0) = \frac{q}{\pi \sigma_y \sigma_z U} \exp \left( -\frac{y^2}{2 \sigma_y^2} \right) \exp \left( -\frac{z^2}{2 \sigma_z^2} \right)
\]

Note that in Turner’s formulation the \( y \)-axis constitutes the plume axis (i.e., \( y_0 = 0 \) in our notation). Allowing the possibility of a coordinate translation gives

\[
C(x, y, z) = \frac{q}{\pi \sigma_y \sigma_z U} \exp \left( -\frac{(y - y_0)^2}{2 \sigma_y^2} - \frac{z^2}{2 \sigma_z^2} \right)
\]

which is our equation (1).

We did not explicitly refer to reflections in the text - we apologise to the reviewer for any unnecessary confusion. We have now changed the text to: (extra text in red):

“The fundamental assumption is that the plume dispersion may be modelled by a Gaussian distribution. With the source at the surface, (\( z = 0 \)), (see, e.g., equation 2.1 from Turner 1994):
where $q$ is the source strength (mass emission rate) of the methane leak, $C(x,y,z)$ is the molar concentration which varies in the $x$ (downwind), $y$ (cross-wind) and $z$ (vertical) directions and $U$ is the mean prevailing wind speed. The $\sigma_y^2$ and $\sigma_z^2$ terms are the mean squared distances of the plume spread in the cross-wind and vertical directions (both growing by dispersion with down-wind distance). The source is fixed at $x = 0$. Note that this form of the equation includes reflection from the surface.”

Fig. 1: “could do with more description” We have added the following text the figure caption:

The left map shows the location of the field in the North Sea, with the red rectangle shown on the right panel. Thw black dot indicates the location of the Elgin platform.

Fig. 2. “The units on Fig. 2 are non-intuitive…”

We have now changed this figure to be more visually appealing: the number of coloured contours has been increased and the NCAR logo has been removed. In addition, the contours now use PPB (converted via the molecular weight of CH$_4$).

Fig. 3: “couldn’t find a description of what “Shearwater”, “Jasmine”, “Judy”, or “Franklin” were”

We have added the following sentence to the figure caption:

The different platforms in the area (Elgin, Shearwater, Franklin, Judy and Jasmine) are shown by the different colour circles.

Fig. 4: “Pretty hard to see what’s going on in this figure, there’s a lot of whitespace that’s taken up by the legend (almost half of each panel is blank).”

We feel that we should leave this figure as it is. We wanted to plot all the runs on the same scale to allow the reader to easily see the changes in methane enhancement on the different flights and feel this is the best way to do this.