Flux calculation of short turbulent events – comparison of three methods

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Abstract. The eddy covariance method is commonly used to calculate vertical turbulent exchange fluxes between ecosystems and the atmosphere. Besides other assumptions, it requires steady state flow conditions. If this requirement is not fulfilled over the averaging interval of, e.g., 30 min, the fluxes might be mis-calculated. Here two further calculation methods, conditional sampling and wavelet analysis, which do not need the steady state assumption, were implemented and compared to eddy covariance. All fluxes were calculated for 30 min averaging periods, while the wavelet method – using both the Mexican hat and the Morlet wavelet – additionally allowed to obtain a 1 min averaged flux.

The results of all three methods were validated against each other for times with best steady state conditions and well developed turbulence. An excellent agreement of the wavelet results to the eddy covariance reference was found, where the deviations to eddy covariance were on the order of on the order of < 2 % for Morlet as well as < 7 % for Mexican hat and thus within the typical error range of eddy covariance measurements. The conditional sampling flux also showed a very good agreement to the eddy covariance reference, but the occurrence of outliers and the necessary condition of a zero mean vertical wind velocity harmed its general reliability. Using the Mexican hat wavelet flux in a case study, it was possible to locate a nightly short time turbulent event exactly in time, while the Morlet wavelet gave a trustworthy flux over a longer period, e.g., 30 min, under consideration of this short-time event.

At a glance, the Mexican hat wavelet flux offers the possibility of a detailed analysis of non-stationary times, where the classical eddy covariance method fails. Additionally, the Morlet wavelet should be used to provide a trustworthy flux in those 30-minute-periods where eddy covariance led to low quality due to instationarities.

1 Introduction

The eddy covariance technique is a common method to measure vertical turbulent exchange fluxes between ecosystems and the atmosphere. It has the great advantage of being a direct and in situ measurement method (Aubinet et al., 2012) integrating over ecosystem scale without disturbing it significantly. However, eddy covariance requires a few important assumptions to be fulfilled, e.g., steady state flow conditions and horizontal homogeneity. Mainly under conditions of stable stratification and due to a possible violation of the steady state assumption, the fluxes might be mis-calculated by eddy covariance, which needs
an averaging period of about 30 minutes to resolve a turbulent flux properly. Typical examples are microfronts or intermittently turbulent flows.

The application of wavelet analysis became popular in geoscience and atmospheric turbulence at the beginning of the 1990s (Farge, 1992; Kumar and Foufoula-Georgiou, 1997; Torrence and Compo, 1998). First they were used to detect jumps of turbulent motions (Mahrt, 1991) or the duration of turbulent events including filtering analysis (Collineau and Brunet, 1993a, b). The application of the spectra of the wavelet coefficient (Collineau and Brunet, 1993b; Treviño and Andreas, 1996) offered also the possibility to determine turbulent fluxes (Katul and Parlange, 1995; Handorf and Foken, 1997). With the availability of wavelet software packages in the last 20 years all these methods became more and more popular, like the detection of turbulent structures (Thomas and Foken, 2005) or turbulent fluxes of aircraft measurements (Strunin and Hiyama, 2004) and at forested sites (Thomas and Foken, 2007). The method was also applied to improve the frequency correction of eddy-covariance measurements (Nordbo and Katul, 2013).

Conditional sampling is furthermore applicable under non-steady conditions and was proposed by Desjardins (1977) as an experimental approach for trace gas measurements. Due to a lack of quick response valve control technology it was impossible to find a practical realization in Desjardin’s days. Today it is available as a mathematical tool for flux calculations in the case of coherent structures (Antonia, 1981; Collineau and Brunet, 1993b; Thomas and Foken, 2007).

Up to now no direct comparisons of the eddy covariance method with results obtained by wavelet analysis and conditional sampling for long-time periods of methane fluxes have been conducted. So the challenge of this paper is not only the comparison of both already often-applied methods in relationship to the eddy-covariance method but also to show with two examples that the quality test for eddy-covariance data (Foken and Wichura, 1996) is a good tool to filter non-steady state time series for which the wavelet or conditional sampling tools are alternative flux calculation methods.

2 Material and Methods

2.1 Data Basis and Instrumentation

The data used for this work was obtained from June to September 2014 at the study site (68.613° N, 161.342° E, 6 m above sea level), which is located 15 km south of the settlement Chersky in the Sakha (Yakutia) Republic, Far Eastern Federal District of Russia and about 150 km south of the East Siberian Sea. It is part of the flat floodplains of the Kolyma river and situated in an area of continuous permafrost. Climatically the area can be described as continental with a dry, warm and short summer from June to August as well as a long, extremely cold winter.

An eddy covariance system was running continuously since July 2013. The measurements were conducted using the heatable 3D sonic anemometer USA-1 (Metek GmbH, Elmshorn, Germany) combined with a closed-path setup, where the inlet of the gas tube was fixed directly below the sonic anemometer. This tube (Eaton Synflex decabon, length 13.8 m, Reynolds number $Re > 2300$) was connected to the gas analyser FGG-24r-EP by Los Gatos Research (Mountain View, California, USA) for $H_2O$, $CO_2$ and $CH_4$, which was installed in a nearby wooden cabin. The concentration expressed as wet mole fraction in the raw data collected by the gas analyser was converted to dry mole fraction immediately, thus the results are independent of
changes in temperature and humidity. The aerodynamic height of the USA-1 was 5.41 m above zero-plane displacement due to the existing tussocks. The tower was supplied with electric power by a fuel powered generator located at the shore of Ambolyka river, a tributary of river Kolyma.

2.2 Data processing and quality control for all methods

The raw data from the sonic anemometer and the closed-path analyser were collected by the software EDDYMEAS (Kolle and Rebmann, 2007) at a sampling rate of 20 Hz, while all other meteorological data was conducted using the CR3000 Micrologger combined with the software LoggerNet (Campbell Scientific Inc., Logan, Utah, USA). Both software programs were running on a personal computer located together with the gas analyser in the wooden cabin. All data refer to local time, where Chersky was covered by Magadan time, i.e. coordinated universal time (UTC) +12 h. The mean local solar noon is UTC + 13 h.

As the present work aimed in a methodological comparison of different flux calculation methods, the early preprocessing for all three methods was done identically using the software TK3 (Mauder and Foken, 2015b), first of all consisting of the conversion from electrical voltage to actual physical units and the detection of spikes. Using the Median Absolute Deviation (MAD) following Hoaglin et al. (2000),

$$MAD = \langle |x_i - \langle x \rangle | \rangle,$$  

(1)

where $\langle x \rangle$ describes the median of $x$, a spike test

$$\langle x \rangle - \frac{q \cdot MAD}{0.6745} \leq x_i \leq \langle x \rangle + \frac{q \cdot MAD}{0.6745}$$

(2)

was conducted, where the threshold value was $q = 7$ and the value 0.675 corresponds to the Gaussian distribution. Values $x_i$ exceeding the given range in expression (2) were labeled as spike, removed and linearly interpolated. Afterwards the 20 Hz concentration and vertical wind speed data was cross-correlated to correct time delays between the sensors. Coordinate rotation was not applied due to the very flat terrain, so over typical time periods for the planar fit rotation (Wilczak et al., 2001) $\varpi = 0$ can be assumed also without rotation.

After these preliminary steps, the covariance was calculated using the three different methods eddy covariance (section 2.3), conditional sampling (section 2.4) and wavelet analysis (section 2.5).

In order to obtain the finalized flux, a number of additional corrections should be applied to the calculated covariance, e.g., the transformation of the measured buoyancy flux into the sensible heat flux (Schotanus et al., 1983) as well as a spectral correction in the high frequency range (Moore, 1986) and the WPL correction (Webb et al., 1980), which accounts for non negligible density fluctuations. As the corrections are identical for all calculation methods, their application was omitted for the present methodological study. It should be noted that these omitted corrections are necessary to obtain the real ecosystem exchange.
2.3 Eddy Covariance Method

The eddy covariance method is based on the turbulent Navier-Stokes equation (Stull, 1988) of mean motion for turbulent flow and allows direct flux measurements, i.e., for flux calculation empirical constants are not necessary (Foken, 2016). So the covariance of the vertical wind speed $w$ and the concentration $c$ can be calculated as

$$\overline{w'c'} = \frac{1}{N-1} \sum_{k=0}^{N-1} [(w_k - \overline{w}) \cdot (c_k - \overline{c})].$$

(3)

Particularly important assumptions are fully developed turbulent flow as well as horizontal homogeneity of the surface and thus the flow field (Foken and Wichura, 1996; Foken et al., 2012).

For this study the program TK3, version 3.11 (Mauder and Foken, 2015a), well-compared to other processing tools (Fratini and Mauder, 2014; Mauder et al., 2008), was used. It conducts the covariance calculation and also allows to apply the data quality tools (Foken and Wichura, 1996) on the results.

2.4 Conditional Sampling

The conditional sampling method – also known as eddy-accumulation – is based on Desjardins (1977), where the covariance $\overline{w'c'}$ of a turbulent flux can be calculated as

$$\overline{w'c'} = \overline{w^+c} + \overline{w^-c} = \left(\overline{w^+ + w^-}\right) \cdot \overline{c} + \overline{w^+c'} + \overline{w^-c'}$$

(4)

with mean vertical wind $\overline{w} = \left(\overline{w^+ + w^-}\right) = 0$. Expressed in words, within an averaging period, e.g., 30 min, the gas concentrations for up- and downwind situations are stored separately and weighted with the associated vertical wind velocity. The flux is then calculated for both storages, which are summed up afterwards over the averaging period. Using data from an eddy covariance tower, the conditional sampling method provides a second possibility for direct flux investigation, while the assumptions to be considered are the same.

In the present study the conditional sampling flux was calculated following Eq. (4), where the mean vertical wind $\overline{w}$ was obtained using the block-averaging method (Finnigan et al., 2003; Rebmann et al., 2012)

$$\overline{w} = \frac{1}{N} \cdot \sum_{n=1}^{N} w_n$$

(5)

in intervals corresponding exactly to eddy covariance of $\Delta t = 30$ min.

2.5 Wavelet Analysis

The wavelet transform allows the decomposition of a time series into the frequencies that represent the signal without losing information about its localisation in time (Percival and Walden, 2008; Torrence and Compo, 1998).

A continuous wavelet transform of a discrete time series $x(t)$ can be written as convolution of $x(t)$

$$T(a,b) = \int_{-\infty}^{\infty} x(t) \cdot \psi^{*}_{a,b}(t) \, dt,$$

(6)
where \( T(a,b) \) is the wavelet coefficient and \( \psi_{a,b}(t) \) is referred to as wavelet function, which depends on scale \( a \) and timeshift \( b \) of a wavelet \( \psi \). If the chosen wavelet is complex-valued, then the complex conjugate \( \psi^*_{a,b}(t) \), denoted by a star sign, is used.

The expression \( T^2(a,b) \) across all times and scales provides the total energy of the time series and the average of the wavelet scalogram \( |T^2(a,b)| \) is used to obtain the wavelet spectrum (Torrence and Compo, 1998)

\[
E_x(j) = \frac{\delta t}{C_\delta} \cdot \frac{1}{N} \sum_{n=0}^{N-1} |T^2(a,b)|
\]  

(7)

over a given number \( N \) of values in the time series, taking the timestep \( \delta t \) and a wavelet-specific reconstruction factor \( C_\delta \) into account. From this it is now possible to obtain the global variance of the time series by integrating over all scales \( j = 0 \) to \( J \)

\[
\sigma_x^2 = \delta j \cdot \sum_{j=0}^{J} \frac{E_x(j)}{a(j)}
\]  

(8)

with \( \delta j \) referring to the spacing between discrete scales and \( J \) being the maximum number of scales. It should be noted that the wavelet scale is not equal to the Fourier period \( \lambda \), i.e., the inverse frequency, but depends on the chosen wavelet \( \psi \). Its choice depends on the intended use. In the present study, the analysis was conducted using both the Mexican hat and the Morlet wavelet. Especially the latter one has been proven to be an appropriate choice for atmospheric turbulence (e.g., Thomas and Foken (2005); Strunin and Hiyama (2004); Terradellas et al. (2001)). While the strengths of the Morlet wavelet are in a very good localisation in the frequency domain, the advantage of the Mexican hat wavelet is on edge detection and provides an excellent resolution in the time domain. Thus it allows an exact localisation of single events in time (e.g., Collineau and Brunet (1993a)).

For two simultaneously recorded timeseries \( x(t) \) and \( y(t) \) the wavelet cross spectrum can now be obtained in analogy to Eq. (7) as

\[
E_{xy}(j) = \frac{\delta t}{C_\delta} \cdot \frac{1}{N} \sum_{n=0}^{N-1} [T_x(a,b) \cdot T_y^*(a,b)]
\]  

(9)

where \( T_y^*(a,b) \) denotes the complex conjugate of the wavelet transform of the second time series \( y(t) \) (Hudgins et al., 1993). Summing up over all scales yields the covariance (Stull, 1988)

\[
\overline{x'y'} = \delta j \cdot \sum_{j=0}^{J} \frac{E_{xy}(j)}{a(j)}
\]  

(10)

for the chosen averaging interval. If the chosen time series \( x \) and \( y \) are the vertical wind velocity \( w \) and a corresponding gas concentration \( c \), the flux \( \overline{w'c'} \) can be calculated now using Eq. (10). Wavelet analysis offers the possibility to calculate fluxes over short averaging times, which are defined by choosing a proper summation interval \( n \) to \( N - 1 \) in Eq. (9). Due to the transformation into scale and time domain, low frequency flux contributions are not neglected; nevertheless it is also possible to include only a subset of frequencies by limiting the summation interval \( j \) to \( J \) in Eq. (10).

As the intention of this study was on short events as well as a comparison to the traditional eddy covariance method, the wavelet cross spectrum was calculated for both averaging intervals \( \Delta t = 1 \) min and \( \Delta t = 30 \) min. In the last step to obtain
the final wavelet flux the cross wavelet spectrum was integrated over the scales following Eq. (10). Calculating the equivalent to the averaging time for the eddy covariance calculation of 30 min, the scale integration interval was set from the smallest equivalent period to 33 and 34 min for Mexican hat and Morlet wavelet, respectively. The difference of 3 and 4 min arises out of the wavelet-dependent calculation of the period \( \lambda \) and the choice of the spacing parameter \( \delta_j \). Here, as a compromise between good resolution in frequency domain and required amount of random access memory (RAM) \( \delta_j \) was set to 0.25.

### 2.6 General Survey of the Flux Investigation Methods

Although the eddy covariance method has been proven as the highly accurate standard, the wavelet analysis allows to neglect two main requirements of eddy covariance: at first, the time averaging can be smaller than 10 to 30 min due to wavelet decomposition in time and frequency domain without ignoring flux contributions in the low-frequency range. Secondly, wavelet transform does not require steady-state conditions, but can also be applied on timeseries containing non-stationary power (e.g. Strunin and Hiyama (2004); Terradellas et al. (2001)). On the other hand, the calculation of fluxes using wavelet transform requires considerably more amount of computational resources, even when a windowed approach is used. In contrast, conditional sampling still requires an averaging interval analogously to eddy covariance, but there is no need to satisfy the steady state condition – provided that \( \bar{w} = 0 \) was chosen absolutely correct, which might be extremely difficult. Table 1 gives a short survey to keep the main points as well as strengths and weaknesses in mind.

### 2.7 Quality Control

In order to compare the three calculation methods, no more corrections were applied, but a second run of the TK3 routine was executed to provide quality assessments. It based on double rotation, spectral correction in the high frequency range (Moore, 1986) as well as a crosswind correction of the sonic acoustic temperature after Schotanus et al. (1983). As the raw concentration data was already converted into dry mole fraction, the WPL correction (Webb et al., 1980) was not applied to the data. This corrected data set was needed to select times with best steady state conditions and well developed turbulence. For the stationarity test, the mean covariance derived from 5 min intervals was compared to the covariance of the whole 30 min interval (Foken and Wichura, 1996) and best conditions were assumed, if the difference was not greater than 30 %. The integral turbulence characteristics (ITC) describe the current state of the atmospheric turbulence integral over the frequency spectrum and can be modelled using parametrisations, based on the concept of flux-variance similarity, and depend on the atmospheric stability (Foken et al., 2004). In the case of a well developed turbulence, the difference of modelled and measured ITC was not greater than 30 %.

### 2.8 Selection of non-steady state events

In order to detect short-time turbulent events, a MAD spike test similar to Papale et al. (2006) using equation (2) was conducted,

where

\[
x_i = (y_i - y_{i-1}) - (y_{i+1} - y_i)
\]  

(11)
parametrises the change $x_i$ in flux $y$ over time. If there is no change in slope from $t_{i-1}$ over $t$ to $t_{i+1}$, then $d_i = 0$. Positive peaks as well as increasing slopes lead to $x_i > 0$, negative peaks and decreasing slopes to $x_i < 0$. Due to its robustness, the median absolute deviation is a very good measure of the variability of a time series and substantially more resilient to outliers than the standard deviation (Hoaglin et al., 2000). The test was applied on the Mexican hat wavelet flux with a time step of $\Delta t = 30\text{min}$. If a value $x_i$ in the time series exceeded the given range in Equation (2), it was detected as an interval containing an event. As the measuring period started in Arctic spring and ended in Arctic autumn, the test was not applied on the whole dataset, but in consecutive steps of 15 days to minimise seasonal influences. A threshold value of $4 \leq q \leq 6$ was found to be suitable to resolve the location of such events in time.

2.9 Validation of the results

For result validation of the methane flux a statistical evaluation using the concept of linear orthogonal regression (Dunn, 2004) was conducted. As the assumption of normally distributed residuals and a homogeneity of their variance, i.e., homoscedasticity, was not fulfilled, the coefficient of determination $R^2$ was obtained using the nonparametric Spearman’s rank correlation coefficient (Hollander and Wolfe, 1973). As the permissibility of further statistics on the linear regression would require a transformation of the data, e.g., using a logarithm or the square root in order to fulfil the above mentioned assumptions, no more tests were conducted. To give anyhow a rough estimate of the maximum standard deviation of the modelled correlation, $\sigma_x$ and $\sigma_y$ was calculated as

$$
\sigma_y = \sqrt{\frac{\sum \epsilon_y^2}{n-2}}, \quad \sigma_x = \sqrt{\frac{\sum \epsilon_x^2}{n-2}},
$$

assuming $x$ and $y$ being the causal (predictor) variable, respectively. The denominator $n-2$ takes the reduction of the degrees of freedom by the two variables $x$ and $y$ into account, while the residuals $\epsilon$ were calculated as

$$
\epsilon_y = y - \hat{y}, \quad \epsilon_x = x - \hat{x}.
$$

3 Results

3.1 Comparison of the methods for steady-state and turbulent conditions

An evaluation of the quality of the calculated results of both newly implemented methods for conditional sampling and wavelet fluxes was necessary to be sure that they are reliable. In this chapter all results were validated against each other for times with best steady state conditions and well developed turbulence as described in detail in chapter 2.3. While the data availability over the whole measuring period was 92.2 %, about 57 % (1292 hours) of the data in the captured time satisfied the stationarity and turbulence requirements mentioned above. All validation results refer to an averaging time of $\Delta t = 30\text{min}$ for all methods as well as to Fourier periods of $\lambda \leq 33\text{min}$ for Mexican hat and $\lambda \leq 34\text{min}$ for Morlet fluxes.
3.1.1 Conditional sampling vs. Eddy covariance

In comparison to wavelet analysis, only for conditional sampling 17 (0.7%) outliers were found and consequently removed by adaption of the MAD test ($q = 5$) from Eq. (2) on the orthogonal residuals. These outliers were found only for eddy covariance fluxes up to $0.5 \text{ nmol mol}^{-1} \text{ m s}^{-1}$ and thus the greater the eddy covariance reference flux, the better the results of both methods coincided.

Besides the found outliers, the very good regression slope of $m = 0.989$ (Fig. 1) and coefficient of determination $R^2 = 0.978$ confirmed a good agreement between eddy covariance and conditional sampling. Unfortunately the occurrence of values with bad quality (outliers) even under best steady state conditions and well developed turbulence makes the general use problematic, but a general dependency between single meteorological parameters and the occurrence of extreme conditional sampling spikes was not found. The reason for these outliers gets explainable, when taking the method’s use of $w$ into account: eddy covariance and wavelet analysis both base on the correlation between $w$ and $c$, for eddy covariance there is the additional requirement of $\bar{w}\bar{c} = 0$ over the averaging period or in the long term (Wilczak et al., 2001). However, in the conditional sampling method $w$ is directly taken into account in Equation (4). In consequence there is a strong dependency on an absolutely correct chosen value for the mean vertical wind $\bar{w}$, where even small inaccuracies lead to a flux bias.

3.1.2 Wavelet analysis vs. Eddy covariance

Comparison of Morlet against Mexican Hat Wavelet Flux

The main difference on the behaviour between the Mexican hat and Morlet wavelet is the excellent resolution in time domain at the first (see also Fig. 4, third panel) and in frequency domain at the second wavelet (Fig. 4, second panel). As the spacing between the discrete wavelet scales, $\delta j = 0.25$, should be chosen small enough, a very good agreement between the results of both wavelets was expected and at last also observed (Fig. 2). The mean regression line has a slope of $m = 0.979$ as well as an intercept of $t = -0.010$ and thus it coincides nearly perfectly with the line through origin of slope 1.

About 99.5% of the variance in the results of each method can be explained by the linear relationship. Theoretically deciding for a predictor variable, the standard deviations for $x$ or $y$ being independent nearly coincide, where $\sigma_y = 0.036$ and $\sigma_x = 0.037 \text{ nmol mol}^{-1} \text{ m s}^{-1}$ for CH$_4$ flux. All in all it can be summarised that – except for a few negligible outliers – the fluxes are almost identical under consideration of the residuals standard deviation.

Comparison of Wavelet Fluxes against Eddy Covariance Flux

In contrast to the comparison of the two wavelet methods, the validation of both against the eddy covariance flux determined the actual quality of the calculated results, because the latter is considered the reference standard in the context of this study. For each wavelet (Fig. 3) a slope of $m \approx 1$ was detected, where the Morlet wavelet showed a closer agreement with the ideal slope (1.023) than the Mexican hat (1.045). The deviations between the eddy covariance and wavelet results were on the order
of < 2 % for Morlet as well as < 7 % for Mexican hat and therefore within the range of the typical error in eddy covariance measurements and processing of about 5 to 10 % (Mauder et al., 2006, 2007b).

To sum up, the method developed and implemented in this study to obtain methane fluxes using wavelet analysis coincided very good with the eddy covariance results under best steady state conditions and well developed turbulence. Both methods resulted in a small, but detectable underestimation of the flux, where the use of the Morlet wavelet marginally showed better results. This is due to its excellent frequency resolution and in agreement with other authors who also applied or recommended the Morlet wavelet on atmospheric turbulence time series (e.g. Farge (1992); Mauder et al. (2007a); Thomas and Foken (2007); Charuchittipan et al. (2014)). In contrast, the Mexican hat flux showed a marginally greater deviation, but is nonetheless within the typical error range of eddy covariance. Thus the Mexican hat wavelet is suitable especially for a high temporal resolution of the flux.

3.2 Case studies

3.2.1 Full developed turbulence

In order to discover a situation under well developed turbulence and best steady state conditions, the afternoon of 23.07.2014 from 13 to 16 o’clock was chosen as a random example (Fig. 4). The atmospheric stratification was unstable and the friction velocity ranged around 0.4 ms\(^{-1}\) with only very low variance over time. Also the mean wind speed was almost constant over time with a mean of 4.7 ms\(^{-1}\), i.e. a gentle breeze coming from North to North-West. The afternoon was sunny with only a few high clouds, thus the absolute value of the short-wave down-welling radiation reached its maximum at 676 Wm\(^{-2}\) in the late noon at 13:30 and decreased afterwards continuously to 590 Wm\(^{-2}\) at the end of the example period. The high solar radiation led to a warming of the surface and therefore to an increasing air temperature caused by the sensible heat flux as well as to decreasing relative humidity over the investigated time interval. The wavelet cross-scalograms did not show any signs of irregularities which could have been caused by sudden events, while eddy covariance and wavelet flux almost perfectly coincided. Referring to the 30 minute average, the Morlet wavelet always showed a greater methane flux by 0.01 to 0.03 nmol mol\(^{-1}\) m s\(^{-1}\) than the Mexican hat. In comparison of the Morlet wavelet flux to eddy covariance, there were only differences by −0.02 to 0.03 nmol mol\(^{-1}\) m s\(^{-1}\) and therefore it can be said, that the Morlet flux resulted in a better accordance than the Mexican hat – this is also in agreement to the general findings in chapter 3.1.2.

In contrast to the very good agreement of wavelet and eddy covariance fluxes, the conditional sampling results showed a non-systematic deviation from the latter flux type by −0.10 to 0.18 nmol mol\(^{-1}\) m s\(^{-1}\). As already discussed, a substantial meteorological reason for that deviations was not found. Assuming a small error of only ±1 \cdot 10^{-4} \text{ms}^{-1} in the correct determination of \(\overline{w}\) (turquoise error bars in Fig. 4, bottom plot) was enough to explain the found variability, i.e., the method is highly sensitive to the correct estimation of the mean vertical wind speed.
3.2.2 Short time turbulent event

Filtering the 1 minute averaged wavelet flux as described in section 2.8, several mostly nocturnal short-time turbulent events were found. One of these occurred in the night from 02. to 03.08.2014 (Fig. 5). It was a clear night with initially only a light breeze with a maximum around 1.5 m s\(^{-1}\), which decreased to a calm situation around 23:30. After that with upcoming turbulence the methane concentration increased rapidly by more than 500 nmol mol\(^{-1}\) around midnight. At 23:59 the 1 minute wavelet flux consequently increased rapidly from 1.9 up to 6.8 nmol mol\(^{-1}\) m s\(^{-1}\) – this is the beginning of the event, which lasted until 0:07. Exactly in the time interval 23:30 to 23:59, where the event begin was detected, the (half hourly calculated) friction velocity \(u_*\) also increased up to 0.1 m s\(^{-1}\). Both methane concentration and friction velocity decreased afterwards.

In the same time during the event the eddy covariance flux quality was determined to be very low following the overall quality flag system by Foken et al. (2004). This is due to the violation of the steady-state assumption (Foken and Wichura, 1996). The conditional sampling flux was nearly equal to the eddy covariance flux, but due to its dependency of the mean vertical wind (chapter 3.1.1), it is not reliable here.

As wavelet analysis does not require steady-state conditions, the obtained wavelet results are the most trustworthy fluxes. The wavelet flux over 30 minutes was about 1.0 nmol mol\(^{-1}\) m s\(^{-1}\) (Morlet) to 1.5 nmol mol\(^{-1}\) m s\(^{-1}\) (Mexican hat) smaller than the eddy covariance result. Using an averaging period of 1 min, the flux using Mexican hat showed greater peaks than the Morlet version. This difference in results between the two wavelets was due to their characteristic properties: the Mexican hat flux allowed an exact localisation of the event in time under consideration of an indistinct resolution in frequency domain. On the other hand the Morlet flux resolves the flux contributions in frequency domain best, but the time domain resolution is not precise. In consequence, only the Mexican hat wavelet was able to resolve the event exactly in time, while the Morlet wavelet results should be more trustworthy in order to obtain the flux balance over a longer time, e.g., 30 minutes.

As this study aims on a methodological comparison, the meteorological and ecological discussion of this event will be presented in a future paper.

4 Conclusions

The aim of the present study was to develop a software, which calculates the flux from 20 Hz wind and methane concentration data in order to resolve and investigate peaks in flux of only short duration within minutes properly. Under best steady state conditions and well developed turbulence it was found that the 30 min averaged results of the developed routine based on wavelet analysis were in very good agreement with eddy covariance. This also implies, that the wavelet results itself might be used as reference flux in future studies. For conditional sampling a high sensitivity regarding the correct choice of \(w\) could be shown, which led to a non negligible number of outliers. Ignoring these spikes, a satisfactory agreement to eddy covariance was determined, but the unsystematic errors made the results critical for detailed investigations. Conditional sampling may be applied for nearly steady state data sets and can be used as a tool to see the effects of periodic coherent structures on the flux as it was done by Thomas and Foken (2007) or to analyze the coefficients for relaxed eddy accumulation (Riederer et al., 2014).
Eddy covariance is the standard method for flux investigation on ecosystem scale. But in the case of short time turbulent events, it typically results in a flux of bad quality due to a violation of the steady-state assumptions over the averaging period. Exactly in such situations, the wavelet method provided a more trustworthy flux, because it does not require steady state conditions. The Mexican hat flux allowed an exact localisation of the event in time, while the Morlet flux resolves the flux contributions in frequency domain best. Therefore, the Mexican hat wavelet flux offers the possibility of a detailed analysis of non-stationary times, where the classical eddy covariance method fails. Additionally, the Morlet wavelet should be used to provide a trustworthy flux in those 30-minute-periods where eddy covariance led to low quality due to instationarities.

In the next stage of this project, we will evaluate the performance of eddy covariance and wavelet methods to detect fluxes under different types of non-steady-state events, which are typically observed during long term flux monitoring campaigns for CH₄. The overall objective here will be to evaluate whether or not a significant portion of CH₄ emissions is missed by the eddy covariance method, because short term events are regularly discarded from the flux budget because of the resulting very low data quality related to non-steady-state conditions.

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References


Figure 1. Scatterplot of conditional sampling against eddy covariance methane flux for times with best steady state conditions and well developed turbulence. The dashed line follows the function $f(x) = x$ and the solid one is the orthogonal regression line.
Figure 2. Scatterplot of Morlet against Mexican hat wavelet methane flux (right) for times with best steady state conditions and well developed turbulence. The dashed line follows the function $f(x) = x$ and the solid one is the orthogonal regression line.
$E_{\text{co}}$ is a function of the wavelet flux, $f(x) = x$. The dashed line follows the function $f(x) = x$ and the solid one is the mean regression line.

Figure 3. Scatterplot of Morlet (left) and Mexican hat wavelet methane flux (right) against eddy covariance for times with best steady state conditions and well developed turbulence. The dashed line follows the function $f(x) = x$ and the solid one is the mean regression line.
Figure 4. Case study of 23.07.2014. The colours in the wavelet cross-scalograms between $w$ and $c$ denote the flux intensity, blue refer to the smallest, green to medium and red to highest methane flux contributions. The cone of influence is outside of the scalogram, i.e., it was not affected by border effects. The error bars for conditional sampling display the range of the result for $\bar{w} \pm 10^{-4} \text{ ms}^{-1}$. 

$w [\text{mol mol}^{-1} \text{ m s}^{-1}]$
Figure 5. Case study of 02./03.08.2014. The colours in the wavelet cross-scalograms between $w$ and $c$ denote the flux intensity, blue refer to the smallest, green to medium and red to highest methane flux contributions. The cone of influence is outside of the scalogram, i.e., it was not affected by border effects. Dashed lines represent wavelet fluxes with an averaging period of 1 min. The error bars for conditional sampling display the range of the result for $\bar{w} \pm 10^{-4}$ ms$^{-1}$.
<table>
<thead>
<tr>
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<th>Eddy Covariance</th>
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<th>Wavelet Analysis</th>
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<tr>
<td>sampling rate</td>
<td>10 – 20 Hz</td>
<td>10 – 20 Hz</td>
<td>10 – 20 Hz</td>
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<td>time resolution of</td>
<td>10 – 60 min</td>
<td>10 – 60 min</td>
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<tr>
<td>calculated flux</td>
<td></td>
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<td>(Nyquist frequency restricts lower limit)</td>
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<td>turbulent conditions</td>
<td>required</td>
<td>required</td>
<td>required</td>
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<tr>
<td>stationarity / steady-state</td>
<td>required</td>
<td>depends on method to obtain ( \overline{m} = 0 )</td>
<td>not necessary</td>
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<tr>
<td>computing requirements</td>
<td>standard personal computer, ready to use processing software available</td>
<td>standard personal computer</td>
<td>memory intensive calculation, RAM ( \geq 8 ) GB recommendable</td>
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<tr>
<td>standard software</td>
<td>available from several sources, e.g., TK3 (Mauder and Foken, 2015b), EddySoft (Kolle and Rebmann, 2007)</td>
<td>not available</td>
<td>not available, but programs to conduct basic wavelet transform already exist, e.g., R-biwavelet (Gouhier, 2014)</td>
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