Additional material addressing reviewer #1’s comment on the correlation terms for Eq. 23 (old numbering, Eq. 26 new numbering):

Yes it is correct that the correlation terms were neglected in this equation. We now use the full expression, with the correlation coefficient included (now Eq. (26)). Theoretically speaking, this is an important correction. Following the reviewer’s suggestion, we also included his suggested approximation, yielding the addition a Sqrt(2) factor in front of the original expression (now Eq. 27).

We would like to add two other pieces of information:

1) Parameterization of the correlation coefficients for use in Eq. 26:
The most accurate numerical result for Eq. 26 is obtained if we parameterize the altitude-dependence of the correlation coefficients. Starting from a value of 1 at the tie-on altitude, this dependence typically decreases exponentially towards 0 as we integrate density downward. The scale-height of this decrease can be determined empirically by running Monte-Carlo experiments. An example is provided in figure 1 (next page). In this example, we ran 200 Monte Carlo simulations of the same atmospheric profile (forward model), where each simulation includes realistic random detection noise (Poisson statistics, all samples independent). A typical 355-nm Rayleigh high-intensity channel with temperature measurements between 30 and 80 km is used here.

In Figure 1, the black solid curve shows one of the 200 corrected signal profiles S (sum of $g^*N$ according to Eq. 14) as a function of altitude (with respect to the tie-on altitude). In this case the tie-on altitude was chosen at a STN ratio of 10, which is 15-20 km below the uppermost valid density point. The dotted red curve shows the standard deviation of S obtained from all 200 MC simulated profiles. The yellow dash-dotted curve shows the uncertainty $u_{SDET}$ obtained experimentally by computing all covariance terms using Eq. 26. The dash-dotted purple curve shows the uncertainty $u_{SDET}$ obtained if we assume full-correlation ($r=1$ for all $z$). The dotted blue curve shows the uncertainty $u_{SDET}$ obtained if we assume no correlation ($r=0$ for all $z$). The dash cyan curve shows the uncertainty $u_{SDET}$ obtained if we assume no correlation ($r=0$ for all $z$), but including the Sqrt(2) factor in Eq. 27. Finally the green dash curve shows the uncertainty $u_{SDET}$ obtained by parametrizing the correlation coefficients as follows:

$$r_{kk'}(k) = \exp \left( c_0 \frac{z(k) - z(k_{TOP})}{H_0} \right) \quad \text{with} \quad k < k_{TOP}, \ H_0=7 \text{ km, and } c_0=2.5 \quad (R1)$$

2) On the actual impact of $u_{SDET}$ (Eq. 26) on $u_{HDEF}$ (Eq. 28):
The actual “total” temperature uncertainty owed to detection noise, as expressed by Eq. 28, is formed of three terms under the square-root: the first and second terms, associated with the density uncertainty, are indeed much larger than the third term, this latter being the term propagated from Eq. 26). Therefore, the use of Eq. 27 with, or without Sqrt(2) yields almost identical results. An example of the magnitudes of those terms are plotted in Figure 2 next page for a typical 355-High intensity Rayleigh channel.