A novel approach for absolute radar calibration

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Abstract

The theoretical framework of a novel approach for absolute radar calibration is presented and its potential analysed by means of synthetic data to lay out a solid basis for future practical application. The method presents the advantage of an absolute calibration with respect to the directly measured reflectivity, without needing a previously calibrated reference device. It requires a setup comprising three radars: two devices oriented towards each other, measuring reflectivity along the same horizontal beam and operating within a strongly attenuated frequency range (e.g. K or X band) and one vertical reflectivity and drop size distribution (DSD) profiler below this connecting line, which is to be calibrated. The absolute determination of the calibration factor is based on attenuation estimates.

Using synthetic, smooth and geometrically idealised data calibration is found to perform best using homogeneous precipitation events with rain rates high enough to ensure a distinct attenuation signal (approx. 30 dBZ). Furthermore, the choice of the interval width (in measuring range gates) around the vertically pointing radar, needed for attenuation estimation, is found to have an impact on the calibration results.

Further analysis is done by means of synthetic data with realistic, inhomogeneous precipitation fields taken from measurements. A calibration factor is calculated for each considered case using the presented method. Based on the distribution of the calculated calibration factors, the most probable value is determined by estimating the mode of a fitted shifted logarithmic normal distribution function. After filtering the data set with respect to rain rate and inhomogeneity and choosing an appropriate length of the considered attenuation path, the estimated uncertainty of the calibration factor is in the order of 1%. Considering stability and accuracy of the method, an interval of 8 range gates on both sides of the vertically pointing radar is most appropriate for calibration.
1 Introduction

In many domains, for example in weather prediction, now-casting, or hydrology accurate rainfall monitoring is an ongoing issue. The most accurate rain rate estimates are commonly obtained by using rain gauges, which are continuously measuring at one point. Even a dense network of these devices only provides point measurements which are not able to describe the high temporal and spatial variability of rainfall events (e.g. Peleg et al., 2013; Pedersen et al., 2010a; Krajewski et al., 2003). Radar networks, such as those from national weather services or smaller ones operated by research institutions, can provide spatially and temporally highly resolved, area-covering rainfall data (e.g. Lengfeld et al., 2014; Trabal et al., 2013; Lim et al., 2011; McLaughlin et al., 2009; Salazar et al., 2009; Chandrasekar and Jayasumana, 2001; Crum et al., 1998) and are already used to improve drainage control and flash flood warning systems (e.g. Picciotti et al., 2013; Quintero et al., 2012; Delrieu et al., 2009; Corral et al., 2008; Einfalt et al., 2004; Creutin and Borga, 2003; Krajewski and Smith, 2002). However, a disadvantage of using weather radar data is the lower accuracy of the retrieved rain rate in comparison to measurements from rain gauges, since rain rates are not measured directly but derived from reflectivity measurements. Beside errors induced by $Z-R$ relations, attenuation, noise, ground clutter, blocking or interferences, one major limiting factor for precise rain rate determination is radar calibration (e.g. Villarini and Krajewski, 2010; Einfalt et al., 2004; Hunter, 1996).

For relative radar calibration it is common practice to compare reflectivity measurements from radars that simultaneously monitor the same rainfall event (e.g. Hunter, 1996). This adjustment only works for radars within a network and do not allow for an absolute comparison of data with other networks or instruments. Furthermore, the retrieved rain rate cannot be used quantitatively. A frequently used approach is a calibration with respect to rain rate using point measurements from rain gauges at ground level (e.g. Sebastianelli et al., 2013; Thorndahl and Rasmussen, 2012; Pedersen et al., 2010b; Jensen and Pedersen, 2005). This implies the disadvantages of point to area
comparison and differences in measuring height. Additionally, the obtained calibration is highly dependent on the chosen $Z-R$ relation (e.g. Ulbrich and Lee, 1999). In order to avoid this source of error, calibration with respect to radar reflectivity is preferable.

Vertically pointing micro rain radars (MRR) allow a comparison of data at same height levels but still, compared measuring volumes are not necessarily equal and micro rain radars also lack an absolute calibration (e.g. van Baelen et al., 2009; Uijlenhoet et al., 2001).

A novel method for absolute calibration using a setup of three radars, performed without previously calibrated reference device, and calibrating with respect to reflectivity is presented here. It requests a radar network setup and takes advantage of the attenuation, which is generally seen as perturbing effect on measurements. The aim of this paper is the theoretical formulation and the proof of concept validation of the presented method, which has not been investigated before.

The analysis of this absolute calibration method focuses on the application on MRRs. Nevertheless, considered instrumental setup and theoretical framework of the method presented in Sect. 2 are applicable to any strongly attenuated radar type. A proof of concept validation is realised in Sect. 3, and validity and potential of the method are analysed further by means of synthetic data presenting realistic measurement structures in Sect. 4. From this study criteria of appropriated rainfall events for calibration are worked out for ensuing utilisation.

## 2 Theoretical framework

The network setup required in order to apply the absolute calibration method presented here is depicted schematically in Fig. 1. Two horizontally oriented radars ($R_1$ and $R_2$) measure along the same connecting line from opposite directions in a certain height. A third, drop size distribution (DSD) profiling device ($R_3$) is positioned below the measuring path in order to provide measurements at one point of the connecting line. For
the sake of simplicity, the focus of the study presented in the following is on the calibration of $R_3$. However, it is straightforward to calibrate $R_1$ and $R_2$ once $R_3$ is calibrated.

Considering the two horizontally oriented radars $R_1$ and $R_2$ (Fig. 1) operate at a strongly attenuated frequency and measure on the same path from opposite directions, the measured reflectivity provided at a point $s$ on the connecting line, $Z_1(s)$ and $Z_2(s)$, can be expressed as follows (e.g. Bringi and Chandrasekar, 2001):

$$Z_1(s) = C_1 \cdot Z(s) \cdot \exp \left(-2 \int_{s_0}^{s} k(s') ds'\right),$$

$$Z_2(s) = C_2 \cdot Z(s) \cdot \exp \left(-2 \int_{s}^{s_{\text{max}}} k(s') ds'\right).$$

(1)

Similarly, the measured reflectivity provided by the vertically pointing radar $R_3$ at a height $z$ is

$$Z_3(z) = C_3 \cdot Z(z) \cdot \exp \left(-2 \int_{0}^{z} k(z') dz'\right).$$

(2)

The measured reflectivity differs from the intrinsic reflectivity $Z(s)$ by the multiplicative calibration factors $C_1$, $C_2$ and $C_3$ comprising device characteristics and by the two-way attenuation, with $k$ the specific attenuation. The latter is given by

$$k(s) = \int_{D_{\text{min}}}^{D_{\text{max}}} N(D, s) \sigma_e(D) \, dD,$$

(3)

where $N(D, s)$ is the drop size distribution (droplet number per unit volume and per unit size interval) dependent on the drop diameter $D$ and $\sigma_e(D)$ the extinction cross section.
The beam of the vertically pointing Doppler radar R₃ crosses the connecting line at s₃, which will be called reference point in the following. In this point, not only the reflectivity but also the discrete DSD \( N_3(D_j, s_3) \) can be derived from the measured Doppler spectra according to the method of Atlas et al. (1973). Analogous to the reflectivity, the relation between measured and intrinsic DSD is

\[
N_3(D_j, s_3) = C_3 \cdot N(D_j, s_3) \cdot \exp \left( -2 \int_0^h k(z')dz' \right),
\]

where \( h \) is the height of the reference point above R₃. Now we determine the measured attenuation using Eq. (3) with discrete size classes \( D_j \)

\[
k_3(s_3) = \sum_{D_{\text{min}}}^{D_{\text{max}}} N_3(D_j, s_3) \sigma_e(D_j) \Delta D_j
\]

Here, \( \Delta D_j \) is the width of the size classes. Replacing the measured DSD \( N_3(D_j, s_3) \) by Eq. (4) yields

\[
k_3(s_3) = \sum_{D_{\text{min}}}^{D_{\text{max}}} C_3 N(D_j, s_3) \exp \left( -2 \int_0^h k(z')dz' \right) \sigma_e(D_j) \Delta D_j
\]

\[
= C_3 \cdot \exp \left( -2 \int_0^h k(z')dz' \right) \cdot \sum_{D_{\text{min}}}^{D_{\text{max}}} N(D_j, s_3) \sigma_e(D_j) \Delta D_j
\]

\[
= C_3 \cdot \exp \left( -2 \int_0^h k(z')dz' \right) \cdot k(s_3).
\]
In the following, assuming homogeneous conditions in the environment of the reference point allows for resolving Eq. (6) for $C_3$:

$$C_3 = \frac{k_3(s_3)}{\exp[-2k_3(s_3)h] \cdot k(s_3)}$$

(7)

While $k_3(s_3)$ is known from Eq. (5), $k(s_3)$ is derived by comparing reflectivity measurements from $R_1$ and $R_2$ along a selected section of the measuring path. Specific attenuation is assumed to be constant along this particular section. The section bounds are located at $s_3 - \Delta s$ and $s_3 + \Delta s$ on both sides of $R_3$. From Eq. (1), the ratio between measured reflectivity at $s_3 - \Delta s$ and measured reflectivity at $s_3 + \Delta s$ for each radar $R_1$ and $R_2$ separately is

$$\frac{Z(s_3 - \Delta s)}{Z(s_3 + \Delta s)} = \frac{Z_1(s_3 - \Delta s)}{Z_1(s_3 + \Delta s)} \cdot \exp(-4k(s_3)\Delta s)$$

(8)

and

$$\frac{Z(s_3 - \Delta s)}{Z(s_3 + \Delta s)} = \frac{Z_2(s_3 - \Delta s)}{Z_2(s_3 + \Delta s)} \cdot \exp(4k(s_3)\Delta s).$$

(9)

Notice that the calibration factors $C_1$ and $C_2$ cancel out at this point, ensuring the absolute determination of the specific attenuation $k(s_3)$ needed for absolute calibration.

Equalising Eqs. (8) and (9) and rearranging terms gives an expression in which only the specific attenuation $k$ remains as a function of known values:

$$\frac{Z_1(s_3 - \Delta s) \cdot Z_2(s_3 + \Delta s)}{Z_1(s_3 + \Delta s) \cdot Z_2(s_3 - \Delta s)} = \exp(8k(s_3)\Delta s)$$

(10)

By assuming a constant attenuation factor $k(s_3)$ along the considered section, the latter can then be expressed by:

$$k(s_3) = \ln \left[ \frac{Z_1(s_3 - \Delta s) \cdot Z_2(s_3 + \Delta s)}{Z_1(s_3 + \Delta s) \cdot Z_2(s_3 - \Delta s)} \right] \cdot \frac{1}{8\Delta s}$$

(11)
Having determined the absolute specific attenuation at \( s_3 \), comparison with the specific attenuation \( k_3(s_3) \) obtained from DSD measurements of \( R_3 \) (Eq. 5) allows for absolute calibration of \( R_3 \). Combining Eqs. (7) and (11) yields the equation for the absolute calibration factor \( C_3 \). The total expression for \( C_3 \) now only depends on measured values. This calibration approach is valid provided that the DSD measured by \( R_3 \) are representative for the DSD along the measuring path between \( s_3 - \Delta s \) and \( s_3 + \Delta s \).

### 3 Proof of concept

In order verify the theory of the presented method, it is evaluated using synthetic data obtained from a forward model generating reflectivity out of a given rain rate. These data represent perfectly calibrated devices for validation purpose, which means calibration factors \( C_1, C_2 \) and \( C_3 \) are implicitly set to 1.0. The measuring path described in Fig. 1 is divided into 31 range gates of \( \Delta r = 200 \text{ m width} \) each and simulated data for \( R_1 \) and \( R_2 \) are discretised accordingly. After the discretisation, \( 1 \leq i \leq 31 \) describes the range gates along the measuring section, starting at \( R_1 \). We also define the interval between \( s_3 - \Delta s \) and \( s_3 + \Delta s \) to be \( (2n + 1)\Delta r \), where \( n \) is the number of considered range gates on both sides of the range gate comprising \( R_3 \).

A rain rate pattern \( R(i) \) is freely defined according to the requirements of the simulated case. The rain rate is assumed to be constant within one range gate. Out of this, the DSD \( N(D_j, i) \) is calculated for each range gate using the Marshall-Palmer standard distribution (Marshall and Palmer, 1948). Drop diameter classes \( D_j \) from 0.15 to 6.5 mm with a class width of 0.05 mm are used here. The theoretical, intrinsic reflectivity \( Z(i) \) is then calculated from DSD and \( D_j \). Finally, attenuated reflectivities \( Z_1(i), Z_2(i) \) and \( Z_3(h) \), describing measurements from \( R_1, R_2 \) and \( R_3 \), are calculated in analogy with Eq. (1). The specific attenuation \( k_3(i) \) required to simulate the attenuated reflectivity is derived from Eq. (3). The extinction cross section is calculated using Mie-theory according to Morrison and Cross (1974), considering droplet flattening.
This study aims at analysing how the calibration accuracy depends on rainfall intensity and structure, and on the width of the interval chosen for determination of specific attenuation $k(s_3)$ around $R_3$, defined by $n$. Synthetic data are simulated for two idealised rainfall patterns (homogeneous rain intensity along (Fig. 2) or featuring a maximum in the middle of (Fig. 3) the measuring path) with 15 different rainfall intensities in each case according to the method described above. For these rainfall patterns calibration is performed using 12 different widths $(2n + 1)\Delta r$, varying $n$ from 1 to 12 range gates on both sides of $R_3$. Hence, the sensitivity study comprises 180 different combinations of rain rates $R(i)$ and interval widths $(2n + 1)\Delta r$. In order to take into account measurement uncertainties, which are inherent to data, calibration is run in a Monte Carlo simulation with 100 repetitions for each combination, assigning random errors to the data for each of the 100 repetitions. For this the simulated radar reflectivity fields are overlaid with a Gaussian distributed noise with a standard deviation of 2 dBZ for each range gate. For both rainfall patterns, mean and standard deviation of the correction factor $C_3^{-1}$ are calculated from the Monte Carlo simulation results.

In a first analysis, a homogeneous rainfall pattern is considered with rainfall intensities varying between 1 and 15 mm h$^{-1}$. Mean correction factors $C_3^{-1}$ (Fig. 4a) are close to the expected value of 1.0 (perfectly calibrated radar), between 0.995 and 1.005 for most of the tested combinations. Higher deviations can be found at short interval widths with $1 \leq n \leq 3$ range gates and for rain rates below 5 mm h$^{-1}$ (corresponding to approximately 30 dBZ) where values range between 0.95 and 1.25. The stability of these results, quantified by the standard deviation of the obtained factors (Fig. 4b), increases towards higher rain rates and larger $(2n + 1)\Delta r$ as the standard deviation decreases. Starting at high values above 2.00 (3.00 for $n = 1$ and lowest rain rate), which indicate a spread of more than twice the expected value of the calibration factor, the standard deviation reaches values below 0.1 for $R \geq 5$ mm h$^{-1}$ and $n \geq 4$. The standard deviation decreases with increasing rain rate and range gate number $n$, reaching down to 0.016. Hence, calibration of $R_3$ is found to be best for high rain rates and large $n$, both having
the effect of producing a clear and detectable attenuation signal along the considered path section required by the calibration method.

The study of the second analysed precipitation pattern, characterised by a maximum value in rain rate above the location of \( R_3 \), investigates possible impacts of heterogeneity on the calibration results. This precipitation field (Fig. 3) is created by using the shape of a normal distribution in order to define a rain rate \( R(i) \) in each range gate from a maximum rain rate. The maximum rain rate (value above \( R_3 \)) is varied between 4.0 and 13.3 mm h\(^{-1}\), creating 15 fields with different rainfall intensities. Figure 5a and b depict the results for averaged correction factor \( C_{-3}^{-1} \) and corresponding standard deviation. The calibration results differ from the one discussed above for homogeneous rain fields. Here, the mean calibration factor is well determined only for high rainfall intensities and small intervals \( (2n+1)\Delta r \) where it lays in the interval between 0.95 and 1.05. For high rain rates and increasing interval width, the correction factor shows a large negative bias with values down to 0.313, which represents an error of almost 70\% in the calibration. The standard deviation is less dependent on rainfall intensity. Stable results (standard deviations below 0.1) are achieved for \( n > 6 \). A slight tendency toward lower standard deviations at higher rain rates is still visible. This obtained bias in the correction factor \( C_{-3}^{-1} \) can be explained by the shape of the precipitation field. Maximum rain rate above \( R_3 \) induces an observed (attenuation corrected) DSD, and consequently attenuation, at the position \( s_3 \) that is not representative for the whole interval \( (2n+1)\Delta r \). Attenuation corrected specific attenuation \( k_3(s_3)\cdot\exp(2k(s_3)h) \) calculated from DSD measurements of \( R_3 \) (Eq. 5) is then higher than specific attenuation \( k(s_3) \) calculated from measurements of \( R_1 \) and \( R_2 \) along the considered interval (Eq. 11), which assumes a constant, averaged specific attenuation. Consequently, correction factor \( C_{-3}^{-1} \) is erroneously found to be smaller than 1.0 (Eq. 7). The opposite effect occurs in the case of a minimum in the rainfall intensity above \( R_3 \).
4 Test on synthetic data with realistic precipitation patterns

Since the presented method is found to be valid when using idealised, smooth precipitation patterns, a further study is realised based on data with realistic rainfall patterns. For this purpose, measured MRR data from a network installed at the Meteorological Observatory Lindenberg (MOL) operated by the German Meteorological Service (DWD) are used in order to create synthetic data with a realistic texture as given by measurements. The network fits the conditions introduced in Sect. 2 and data were recorded between beginning of May and end of June 2013. Within this period 15 rainfall events, comprising 4220 10 s time steps in total, are chosen for testing the calibration method. Reflectivity measurements from the network are set to be the given intrinsic reflectivity from which synthetic, attenuated reflectivity measurements for all three radars are simulated using the procedure described in Sect. 3. All devices are still considered to be perfectly calibrated for this analysis, yielding a correction factor $C_3^{-1}$ of 1.0.

Since two characteristics of reflectivity fields have been detected to be disadvantageous for calibration in Sect. 3 (high heterogeneity and low rain rates along the measuring section), the simulated reflectivity fields are filtered using two parameters in order to remove unsuited cases. The prerequisite for a good calibration is an attenuation effect strong enough to be detected reliably. Therefore, the rainfall intensity along the measuring path has to be high enough to achieve the required signal extinction. Averaged reflectivity along the measuring path is calculated and a threshold is set to 30 dBZ. Furthermore, strong inhomogeneities, an evidence for high noise or disturbances in the measurements, can falsify calibration. Therefore these data are filtered out. This is done by using the texture of the reflectivity $T_{DBZ}$, according to Hubbert et al. (2009):

$$T_{DBZ} = \sum_i \frac{(\text{dBZ}(i) - \text{dBZ}(i - 1))^2}{I} \cdot \frac{1}{I}$$  \hspace{1cm} (12)

$T_{DBZ}$ is an indicator for fluctuation of reflectivity along the path calculated by summing up the squared differences in reflectivity between adjacent range gates $i$. Here, $I$ rep-
represents the total number of range gates considered for the TDBZ calculation, which has to be chosen appropriately. A TDBZ threshold over the considered \( I = 11 \) range gates in the middle of the measuring path is set to 1.4 dBZ. After filtering with averaged rain rate and TDBZ, 3246 suited time steps remain for calibration and are used in the following. After generation and selection of synthetic reflectivity fields, calibration of \( R_3 \) is performed in order to analyse the behaviour of the calibration method when applied to data showing realistic patterns. The calibration is performed once for each time step, with an added random, Gaussian shaped fluctuation with a standard deviation of 2 dBZ which simulates measurement uncertainties. Eleven different intervals \((2n + 1)\Delta r\) are considered, varying \( n \) between 2 and 12 (\( n = 1 \) was found to provide unsatisfying results in Sect. 3).

Calibration using the selected time steps and intervals \((2n + 1)\Delta r\) does not achieve precise results in all cases. Resulting correction factors are spread over a wide range of values. In order to define a method for the determination of the wanted correction factor, distributions of obtained \( C_3^{-1} \) are studied (Fig. 6). For each considered number of range gates \( n \), the calculated correction factors are considered among 20 classes with a width of 0.2. Time steps providing negative calibration factors are removed, resulting in a different number \( N \) of remaining calibration results for the eleven different interval widths. Those negative results have no physical meaning, as corrected reflectivity fields would then also appear to be negative (Eq. 2). According to Eq. (7) negative correction factors appear if \( k(s_3) \leq 0 \). Since this study is based on synthetic data, as described above, non-physical, negative specific attenuation values are an artefact of the added random noise creating strong inhomogeneities and missed by the TDBZ filter. These cases should not be considered. Distributions of the correction factor have their maximum within the class including 1.0 (except for \( n = 10 \), which is the expected correction factor for a perfectly calibrated radar, and are positively skewed. As discrete distributions only allow for the estimation of a median within the given resolution, a shifted logarithmic normal distribution function is fitted to the discrete distribution in order to provide more precise results. The most probable value of the wanted correction factor...
is given by the mode of the distribution function, thus the most frequently calculated value. In order to describe the width of the distribution, which describes the accuracy of calibration, the interquartile range (describing the distance between 25 and 75% percentile) is also considered (Table 1). Obtained distribution functions have the lowest interquartile range for $4 \leq n \leq 8$, indicating the most stable results for these settings. The widest spread of the calculated correction factor is found using $n = 2$. The error in the estimation of the correction factor, knowing the true value is 1.0, reaches 11% for $n = 2$ and is lowest for $n \geq 8$ where $C_3^{-1}$ is determined by the mode with an error of only 1%. It is especially exactly 1.0 for $n = 9$. These results suggest the possibility to achieve satisfactory calibration results when applying the presented calibration method to carefully chosen data and settings. Since a considered interval $(2n + 1)\Delta r$ with $4 \leq n \leq 8$ yields most stable results, and most accurate results are achieved with $n \geq 8$, $n = 8$ is possibly the most appropriate setting for calibration.

5 Conclusions

A novel method for absolute radar calibration with respect to reflectivity and without reference device is analysed in this study. In order to describe the potential of the novel calibration method, a sensitivity analysis is performed. The method is first tested using synthetic data with idealised, smooth precipitation patterns to prove the validity of the concept. Homogeneous and Gaussian shaped precipitation patterns are analysed and calibration performed considering simulated data with a measuring uncertainty of 2 dBZ. Furthermore, different rainfall intensities and interval widths $(2n + 1)\Delta r$ for determination of the attenuation $k(s_3)$ are taken into account. Calibration using homogeneous precipitation patterns yields precise results. Mean correction factor $C_3^{-1}$ for $R_3$ takes values between 0.995 and 1.005, except from calibration at rain rates below 5 mm h$^{-1}$ and with $n \leq 3$. This represents an error of 0.5% in the determination of the true calibration factor, which is 1.0 for a perfectly calibrated radar as simulated in this analysis. The standard deviation, expressing the stability of the procedure, stays be-
low 10 % for $R \geq 5\text{mm h}^{-1}$ and $n \geq 4$. It decreases towards higher rain rates and larger interval widths reaching down to under 2 %. Gaussian shaped precipitation patterns, featuring a maximum in rainfall intensity above $R_3$, reveal one weakness of the method. Due to higher rain rates above $R_3$, its measurements are erroneously corrected toward lower reflectivity values, inducing a negative bias in the calculated correction factor. This bias is stronger the higher the rain rate and the larger the interval along which the attenuation is determined. Since a minimum in rainfall intensity above $R_3$ is likely to occur as often as a maximum, inducing a positive bias, this effect will cancel out when averaging over rainfall events.

The theoretical validity of the presented absolute calibration method has been proved for adequate precipitation patterns. Ideal cases are preferably homogeneous, intense rainfall along the measuring path, leading to attenuation strong enough to be determined reliably.

Since these first promising results are obtained focusing on idealised synthetic data, a further study is done with synthetic data featuring realistic structures. Those structures are taken from reflectivity measurements from a real deployment of MRRs following the concept introduced in Sect. 2. Considered time steps are filtered using averaged reflectivity along the measuring path and texture of the reflectivity field TDBZ in order to guarantee high attenuation and smooth measurement structures. Calibration over a 4220 sample of filtered time steps of reflectivity measurements leads to distributions of calculated correction factors which can be described using shifted logarithmic normal distribution functions. These functions are fitted to the obtained results and the mode is calculated in order to describe the searched correction factor for each considered $n$. Here, the calibration results vary between 0.89 and 1.01, hence up to 11 % error. Best results are achieved using $n = 8$.

This study shows fundamental analysis of a novel method for absolute radar calibration in a network and proves its theoretical validity. The presented method could offer great opportunities for absolute calibration of radar networks operating in strongly attenuated frequency ranges (e.g. K and X band), providing accurate and comparable
data for application. Future analysis applying the method on selected network data, including comparison with reference devices and established calibration methods, have to be performed in order to prove its applicability in practice.

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References


Table 1. Mode, 25 and 75% percentiles and interquartile range calculated from the logarithmic normal distribution functions fitted to the results for $C_3^{-1}$ for $2 \leq n \leq 12$ (rounded values). Calibration performed using synthetic data with realistic precipitation structures.

<table>
<thead>
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<th>$n = 2$</th>
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Figure 1. Schematic network setup including horizontally oriented radars $R_1$ and $R_2$ and vertically oriented radar $R_3$ (top) and schematic reflectivity measurements for $R_1$ and $R_2$ along the connecting line (bottom).
Figure 2. Idealised synthetic data simulated along the measuring path. Homogeneous precipitation pattern with a rainfall intensity of 15 mm h\(^{-1}\) (top) and corresponding reflectivity fields simulated using forward operator (bottom). Intrinsic reflectivity is shown in green and simulated, attenuated reflectivity in red for \(R_1\) and blue for \(R_2\). The vertical black line marks the position of \(R_3\).
Figure 3. In analogy with Fig. 2, rain rate and simulated reflectivity fields for a precipitation pattern with a maximum intensity above $R_3$ (maximum rain rate is $13.3 \text{ mm h}^{-1}$).
Figure 4. Mean (a) and standard deviation (b) of the correction factor $C_3^{-1}$ for homogeneous precipitation patterns, calculated from Monte Carlo simulations including 100 repetitions with random measuring error. The Monte Carlo simulations are performed for combinations of 15 different intensities of the homogeneous precipitation field and 12 considered interval widths $(2n + 1)\Delta r$ for attenuation determination above $R_3$. 
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Figure 5. Mean (a) and standard deviation (b) of the correction factor $C_3^{-1}$ for Gaussian shaped rain fields, calculated from Monte Carlo simulations including 100 repetitions with random measuring error. The Monte Carlo simulations are performed for combinations of 15 different rainfall intensities and 12 considered interval widths $(2n+1)\Delta r$ for attenuation determination above $R_3$. 

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Figure 6. Distributions of the correction factor $C_3^{-1}$ calculated using synthetic data with realistic precipitation patterns and fitted logarithmic normal distribution functions for six values of $n$. The total amount of considered time steps is indicated by $N$. 