Interactive comment on “Retrieving aerosol height from the oxygen A band: a fast forward operator and sensitivity study concerning spectral resolution, instrumental noise, and surface inhomogeneity” by A. Hollstein and J. Fischer

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The authors present a study of the impact of factors like spectral resolution, instrumental noise and surface inhomogeneity on the retrieval of aerosol type, optical thickness and height. These are important considerations that should inform the development of appropriate aerosol retrieval algorithms for different hyperspectral instruments. The results obtained are thus of high interest and deserve publication.

The authors make adept use of principal component analysis to allow efficient handling of hyperspectral measurements. They provide scatter plots of Jacobians of measured
radiances with respect to retrieval parameters, which highlight the dissimilar dependence of different parts of the spectrum on the aerosol optical thickness and height, making them easy to distinguish from each other using the O₂ A-band, whereas the opposite is seen to hold for AOT and surface reflectance, indicating a high level of correlation which makes their individual effects difficult to separate.

The authors obtain the very interesting that height retrievals depend on a good SNR but are more robust than AOT retrievals when the underlying surface reflectance and/or aerosol type are unknown. This result becomes intuitive given the above-mentioned scatter plots.

While their results and analysis are of high value, it would be advisable to improve the quality of the presentation. In the following, some suggestions are made to this effect:

Typos and Grammar:

1. page 10512, line 11: then → than
2. page 10513, line 2: ascends → ascents
3. Page 10515, line 24: as → than
4. Page 10517, line 4: consists → consist
5. Page 10517, line 17: then → than
6. Page 10517, line 21: and is by a compression factor \( n_\lambda/n_p \) smaller than Y → and is smaller than Y by a compression factor \( n_\lambda/n_p \)
7. Page 10517, line 27: computed → done (double use of “computed” in the same sentence)
8. Page 10520, line 7: sub sample → sub-sample
9. Page 10522, line 9 (and throughout the text/Figure captions): Jacobean → Jacobian

References:

1. Page 10515, Section 2: Include Sanghavi et al. 2012 (Retrieval of the optical depth and vertical distribution of particulate scatterers in the atmosphere using O\textsubscript{2} A- and B-band SCIAMACHY observations over Kanpur: a case study) in addition to Boesch et al. 2006 as a retrieval method with direct use of an RTM as forward model.

2. Page 10519, Section 3: Why has the log-normal parametrization been chosen? Again, cite Sanghavi et al. 2012 (Retrieval of the optical depth and vertical distribution of particulate scatterers in the atmosphere using O\textsubscript{2} A- and B-band SCIAMACHY observations over Kanpur: a case study) as a previous instance of the use of this parametrization.

Content:

1. Page 10515, line 23: Remove “locally” in front of nonlinear behaviour of solutions (redundant)

2. Page 10517, line 10: two instances where “orthogonal” is used instead of “orthonormal”. Suggest orthogonal → orthonormal

3. Page 10519, Section 3, Eq. 2: Include a factor \(1/\sqrt{2\pi}\) on the right hand side

4. Page 10520, Section 3, Eq. 3: Remind the reader that the variables on the right hand side have been discussed in Section 2. Re-describe the variables \(\hat{y}_i\) and \(\tilde{y}_i\) briefly.
5. Page 10521, line 4: Explain what is meant by a “neutral” aerosol type. Provide microphysical parameters (complex refractive indices, size distribution parameters and mode fraction of all modes considered) for all aerosol types considered, preferably in the form of a Table.

6. Page 10521, line 6: extinction → extinction cross-sections

7. Page 10521, line 7/Fig 5: It is difficult to understand why the aerosol phase functions are considered dependent on the AOT. Please clarify.

8. Page 10521, last paragraph referring to Fig. 6 and 7: It is difficult at first sight to understand that the spectral wavelength is the common parameter for the scatter plots. Please make explicit mention of this. Before Figures 6 and 7 are introduced, it is advisable to show the actual partial derivatives (∂I/∂τ, ∂I/∂α and ∂I/∂h) plotted against wavelength for at least one of the aerosol types considered.


10. Page 10522, 2nd last paragraph: Simulated measurements are discussed here, but a definition of a synthetic measurement is only introduced later in Section 5. Please reorder the sequence, so that the discussion can follow the introduction.

11. Page 10522, last paragraph: These statements appear to contradict each other and may need further refinement. For example: how does the asymptotic solution described here work when the step size is large enough to cause a parameter to exceed the bounds of your lookup table by more than a factor of 2? Is it then successively divided by two until it falls within the expected bounds? If that is the case it should be mentioned explicitly. Also, the first line of the paragraph should make it clear that the lmder routine is only nominally implemented for an unbound problem.
12. Figures 9 and 10: Artifacts due to finite resolution of the LUT are still present. Please make mention of this when discussing the results, so that the reader is aware of potential bias due to the artifacts.

13. Figures 9 and 10: Define $tp$ in the text making use of an appropriate equation to show its relation to individual microphysical parameters.

14. Figures 11 and 12: the green and cyan lines are almost indistinguishable. If possible, replace the color scheme with one that has more contrast.

15. Figure 13: Color scheme does not match that indicated in the caption. Again, try to use a color scheme with more contrast: violet (?) and black(?) are almost indistinguishable.
Abstract

Hyperspectral radiance measurements in the Oxygen A band are sensitive to the vertical distribution of atmospheric scatterers, which in principle allows to retrieve aerosol height from future instruments like TROPOMI, OCO2, FLEX, and CarbonSat. Discussed in this paper is a fast and flexible forward operator for the simulation of hyperspectral radiances in the Oxygen A band and, based on this scheme, a sensitivity study about the inversion quality of aerosol optical thickness, aerosol mean height, and aerosol type. The forward operator is based on a lookup table with efficient data compression based on principal component analysis. Linear interpolation and computation of partial derivatives is performed in the much smaller space of expansion coefficients rather than wavelength. Thus, this approach is computationally fast and at the same time memory efficient. The sensitivity study explores the impact of instrument design on the retrieval of aerosol optical thickness and aerosol height. Considered are signal to noise ratio, spectral resolution, and spectral sampling. Also taken into account are surface inhomogeneities and variations of the aerosol type.
1 Introduction

Multiple fields of research can benefit from an accurate and reliable aerosol height product. Among others are atmospheric sciences, where aerosol vertical distribution and interaction with clouds and radiation is discussed (e.g. Chin et al., 2009; Lohmann and Feichter, 2005) as well as long range aerosol transport (e.g. Betzer et al., 1988; Andreae, 1983) and source attribution (e.g. McConnell et al., 2007; Clarke and Noone, 1985), human health (e.g. Nel, 2005; Harrison and Yin, 2000; Seaton et al., 1995; Pöschl, 2005), and in remote sensing of the atmosphere, where its effect on the retrieval of total aerosol optical thickness is discussed (e.g. Quijano et al., 2000; Duforêt et al., 2007; McClain, 2009; Muller et al., 2007).

LIDAR measurements (e.g. using EARLINET (Amodeo et al., 2007)), airborne experiments (e.g. Zieger et al., 2007), or balloon ascents (e.g. Rosen et al., 1975) can be used to derive aerosol vertical profiles on local scales, while the backscatter LIDAR CALIOP onboard the satellite CALIPSO (Winker et al., 2009) is currently the only instrument which provides information on a global scale. A limitation of CALIPSO measurements is their sparse spatial and temporal resolution (Winker et al., 2010; Amiridis et al., 2013), which could be improved drastically by deriving aerosol height directly from passive imaging instruments.

Attempts to use the Oxygen absorption bands for an aerosol height retrieval have been published in the past. For a case study, Duforêt et al. (2007) exploited MERIS and POLDER data to derive aerosol height over oceans from reflectance ratios of channels inside and outside the O_2A band. Sanghavi et al. (2012) discussed to use the O_2A and O_2B band to derive aerosol vertical distribution from SCIAMACHY data and applied the technique to a scene for Kanpur (India). However, for aerosols over land no operational data product exists to our knowledge, although hyperspectral measurements within the Oxygen A band were and are performed by operational instruments such as SCIAMACHY, GOSAT, GOME, and GOME2. However, it is possible to derive the absorbing aerosol index (Torres et al., 1998; De Graaf et al., 2005) from such type of instruments, which is among other aerosol parameters also sensitive to aerosol height, but does not retrieve quantitative aerosol vertical distribution parameters.

In the near and not too distant future, hyperspectral measurements within the Oxygen A band will become widely available from instruments such as OCO2 (Haring et al., 2004; Crisp and Johnson, 2005), TROPOMI (Veefkind et al., 2012), Sentinel-4 (ESA, 2012), Sentinel-5 (ESA, 2012), or if selected, ESA Earth Explorer (Bézy et al., 2008; Meijer et al., 2012) missions like FLEX (Clissold, 2008; Rascher et al., 2008; Stoll, 2003) or CarbonSat (Velazco et al., 2011). Operational aerosol height products are at least planned for TROPOMI onboard the Sentinel 5 precursor (Veefkind et al., 2012; Sanders and de Haan, 2013) and for the Sentinels 4 and 5 (ESA, 2012). The aim of these products is to distinguish between aerosols in the planetary boundary layer and the free troposphere with desirably estimation of aerosol type, e.g. to constrain surface concentrations of particulate matter (ESA, 2012). The specifications of these instru-
ments vary widely with respect to spectral resolution, spatial resolution, temporal resolution, and signal to noise ratio (SNR); and all mission design parameters and each of those mission design parameter might have an impact on a possible retrieval of aerosol height. Sanders and de Haan (2013) discussed the possible retrieval accuracy of aerosol height by propagating measurement and a priori errors trough a locally linearized radiative transfer model using the framework of optimal estimation by Rodgers (2000). From the perspective of an operational retrieval, a positive result for the retrieval error with respect to given user requirements fulfills only a necessary condition. A real-world retrieval must in addition converge robustly, treat multiple minima in the used cost function, deal with deviation of the real and the model atmosphere, and needs to be computationally fast enough to process and possibly reprocess large amounts of hyperspectral radiance data.

Aim of this paper is to propose and present the design and present an implementation of a fast and efficient forward operator for the simulation of hyperspectral top of atmosphere radiances in the Oxygen A band, which is based on accurate radiative transfer simulations. It was applied in a simple inversion scheme, which can represent a generic real world retrieval algorithm. The forward operator is based on upon linear interpolation within a lookup table which was produced with efficient data compression based on using principal component analysis. Linear interpolation and computation of partial derivatives is within this lookup table can then be performed in the much smaller space of expansion coefficients, which makes this approach computationally fast and at the same time memory efficient.

The generality of such a fast forward operator allows to employ sophisticated optimal estimation techniques, as well as simple curve fitting techniques for the retrieval of forward operator parameters from measurements. Since the approach is computational very efficient, it can be applied to operational retrievals, where in general huge amounts of data have to be processed with strict constraints on computational time. Here, the fast forward operator is applied in a simple inversion scheme using a curve fitting technique, which could represent a generic retrieval algorithm for the retrieval of aerosol height.

The fast forward operator is described in Section 2, while Section 3 and 5 cover the setup of the sensitivity study and the inversion scheme. The applied instrument error model, and the generation of synthetic measurements is introduced in Section 4. The inversion scheme is described in Section 5 and the 5 and the the results of the sensitivity study are discussed in Section 6.

2 Fast Forward Operator

The following discussion is based on the assumption, that the inversion of radiative transfer simulations for a complex atmosphere is the most suitable path approach to retrieve aerosol height. Radiative transfer simulations numerically map an atmospheric state, which includes among others the quantities of interest, with simulated measurements. An inversion operator, which could include additional prior knowledge is then can be used as retrieval operator. This assumption is emphasized, since this approach requires in general complex modeling, is prone to errors, requires precise knowledge about the optical properties of the atmosphere, the surface, and the aerosols, and can be computationally very demanding. To our best knowledge, no simpler approach for the retrieval of aerosol height exists up to now, as it was found e.g. for fluorescence emitted by vegetation, which can be retrieved using Fraunhofer lines without relying on radiative transfer (e.g. Frankenberg et al. 2011, Jooper et al. 2011).

The design of the fast forward operator was mainly driven by considerations about computational speed on standard computer hardware and simulation accuracy. This leads either to the use of the radiative transfer model directly as forward model (e.g. compare the OCO2 retrieval by Bosch et al. 2006, e.g. compare the OCO2 retrieval by Bosch et al. 2006 or the retrieval of aerosol height from SCIAMACHY by Sanghavi et al. 2012), or to use the classical approach of a forward operator based on interpolation within a lookup table populated by radiative transfer calculations. The following discussion could also be based on neural networks, but the interpolation approach was chosen since its behavior is easier to understand and does not depend on tuned neural network parameters.

Using a full scale radiative transfer model as forward operator entails some inherent advantages, as one does not rely on (linear) interpolation, which can introduce errors due to the locally nonlinear behavior of solutions of the radiative transfer equation. However, this approach will always be much more demanding computationally as computationally demanding than a simple lookup table interpolation and thus might be unsuitable for application on standard computer hardware systems. A possible solution is to speed up the radiative transfer, which in general sacrifices simulation accuracy, which could cancel some of the advantages of this approach. On the other hand, populating a high dimensional lookup table with hyperspectral radiative transfer simulations is computationally expensive. Also, computationally expensive - especially if more and more state parameters are to be included. However, for the case of the Oxygen A band, this obstacle can be lifted by exploiting the high correlation of the spectrum with respect to variations of the atmospheric state. Nathraj et al. (2005) were using principal component analysis to speed up the calculation of optical parameters for a radiative transfer model, while Hollstein and Lindstedt (2013) have presented an approach based on principal component analyses, which can speed up the population of the lookup table by one or more orders of magnitude. Following section, the approach is described in detail.
lowing the latter approach, a higher speedup is achieved for larger lookup tables. Hence, the computational aspect of filling up a potentially large lookup table can be considered as very well under control.

To summarize this approach: It was shown explicitly for the Oxygen A band, that a comparably small, randomly selected subset of spectra is sufficient to compute a set of principal components, which can be used to reconstruct the total lookup table, where the reconstruction accuracy is a function of the randomly selected subset of spectra and the number of used principal components. Then, Hollstein and Lindstrot (2013) presented an algorithm, which makes it possible to compute the expansion coefficients of a certain spectrum by simulating only a small subset of spectral channels. This implies, that for the Oxygen A band, a relatively small number of spectral channels is sufficient to reconstruct the hyperspectral simulation, thus leading to an enormous speedup. This approach makes it feasible to increase the allowed computation time of the radiative transfer, e.g. to increase simulation accuracy by using higher vertical and spatial resolution or by taking 3D effects into account.

Next to populating the lookup table, its possible huge size can become a major obstacle, especially for hyperspectral applications. As a consequence, the proposed lookup table for the fast forward operator is also exploits a data reduction technique based on principal component analysis. In this way, It is used to achieve effective data compression is used to reduce the size of the lookup table and also to gain computational speedups for the interpolation within that table.

The approach consists of two tables, one which stores the expansion coefficients of the simulated spectra with respect to the corresponding atmospheric state and a second one for the used principal components. Formally: let \( x \in \mathbb{R}^{n_x} \) be a state vector with \( n_x \) free parameters for the radiative transfer model \( RT(x) = y \in \mathbb{R}^{n_y} \), where \( y \) is the simulated spectrum and \( n_y \) is the number of simulated spectral channels. A generic lookup table would then consist of a possibly large set with \( n \) states: \( X = (x_1, \ldots, x_n) \) and a related set of simulated spectra \( Y = (y_1, \ldots, y_n) \). Principal component analysis can be used to derive a number of \( n_P \ll n_y \ll n_x \) principal components \( p_i \in \mathbb{R}^{n_y}, i = 1, \ldots, n_P \), which can be combined in the principal component matrix \( P_{n_P} \in \mathbb{R}^{n_P \times n_y} \) with \([P_{n_P}]_{ij} = [p_i]_j\). The expansion coefficients for each spectrum \( c_i = P \times y_i, i = 1, \ldots, n \) can be computed such that a reconstructed spectrum \( \tilde{y}_i \) can be expressed as \( \tilde{y}_i = P^{-1} \times c_i = P^T \times c_i \). The matrix \( P_{n_P} \) is orthogonal orthonormal since the \( p_i \) are pairwise orthogonal, orthonormal, such that \( P^{-1} = P^T \). The reconstruction residual \( \epsilon_i = |y_i - \tilde{y}_i|, i = 1, \ldots, n \) for a single spectrum \( y_i \) generally depends on the number of principal components and the reconstructed spectrum, while the mean reconstruction residual with respect to the total lookup table \( \epsilon = mean(\epsilon_1, \ldots, \epsilon_n) \) only depends on the number of principal components.

In general and with proper computation of \( P_{n_P} \), both residuals are strongly decreasing with increasing number of principal components, thus \( n_P \) can be chosen such that the mean reconstruction error is sufficiently smaller than the measurement error of a possible instrument. As a result, the large table \( Y \), which contains simulated spectra can be replaced by the principal component matrix \( P_{n_P} \), and a table \( C = (c_1, \ldots, c_n) \) with \( c_i = P_{n_P} \times y_i \), which contains only the expansion coefficients for each spectrum and is smaller than \( Y \) by a compression factor \( \frac{n_y}{n_P} \), smaller than \( Y \).

To save large amounts of computation time for building the lookup table, the \( c_i \) could be computed using the approach presented by Hollstein and Lindstrot (2013). The lookup table \( C \) should be small enough to fit easily in the main memory of modern PCs. Not only does this approach save main memory when compared with an uncompressed table, it also saves much computation time since the interpolation and computation of partial derivatives can be computed in the space spanned by the expansion coefficients. This is possible since the compression from the principal component analysis is linear, thus compression and interpolation can be interchanged.

An interpolation scheme \( IT(X, C, x) = c \) in the expansion coefficient space can be implemented from which the desired interpolated spectrum space can be computed by multiplying with the principal component matrix:

\[
y(x) = P_{n_P}^T \times IT(X, C, x). \tag{1}
\]

This approach can be implemented to be computationally efficient and enables a simple path to exploit computation on modern GPGPUs. In an initialization step, the lookup table \( C \) must be copied to the memory of the GPGPU, which is the slowest part, then very little data transfer is needed since for a single interpolation only the two small vectors \( x \) and \( c \) need to be transferred from and to the main memory, where the final expansion to the desired spectra could be computed. The spectral resolution of the simulations can be adjusted easily and with little extra computational cost. When the original simulations were performed with adequate spectral resolution, it is for many instruments sufficient to convolve the original principal components with the desired appropriate response functions to setup a fast forward operator for a different instrument. This is numerically cheap, as the number of principal components is in general not too large and can be performed on the fly when the program is called. Similarly, spectral shifts as they occur e.g. for TANSO-FTS onboard GOSAT can be corrected by convolving the principal components with response functions which take the spectral shift into account. Further speedup in an inversion scheme can be achieved by selecting the number of used principal components with respect to the current value of the cost function. In the first steps of an iterative optimization scheme, it might be sufficient to reconstruct the spectra using only a few principal components, since it is sufficient to keep the reconstruction error well below the difference of actual measurement and simulation. The number of princi-
pal components can then be increased with decreasing cost function value to improve the reconstruction quality.

3 Synthetic Study Setup

Radiative transfer simulations were performed using the MOMO radiative transfer model (see [Fell and Fischer 2001] [Hollstein and Fischer 2012]) which is a matrix operator model widely used at Freie Universität Berlin. Gaseous absorption was computed using line parameters from the HITRAN spectral database [Rothman et al. 2009] and a modified scheme to compute the k-distribution [Bennartz and Fischer 2000]. [Lacis and Oinas 1991]; Fu and Liou

The parameter grid for the lookup table is shown in Table 1. The variation of the atmospheric state includes surface pressure, aerosol optical thickness, aerosol mean height, aerosol type, surface reflectance, and the viewing geometry. The full parameter range of the lookup table space is used throughout this study. The surface reflectance spectrum is assumed to be a linear function and is modeled using a reflectance value at 755nm and at 780nm. The normalized aerosol vertical distribution $v(h, \mu, \sigma)$ with respect to height above the surface $h$ and mean height $\mu$ is modeled using a log normal distribution with a width parameter of $\sigma = 1.1$, which represents a narrow layer-like distribution:

$$v(h, \mu, \sigma) = \frac{1}{h \cdot \log(\sigma)} \frac{1}{\sqrt{2\pi 2 \log(\sigma)}} \exp\left(-\frac{(\log(h) - \log(\mu))^2}{2\log(\sigma)^2}\right).$$

The log normal model can represent a wide range of possible aerosol vertical distributions and a similar model was also used by Song et al. (2012) in their previously cited case study.

As a result of this setup, the analysis is based on the retrieval of a single aerosol layer when only a single aerosol layer is present. A constant temperature profile is assumed throughout this discussion, since the actual profile should be given as background information. Introducing the temperature profile into this framework poses no specific difficulties and could be easily implemented as proposed by Lindstroth and Preusker (2012).

Several randomly selected spectra at three spectral resolutions are shown in Figure 1. All spectra within the lookup table were fully simulated and could be used to compute the principal components, as well as evaluating the reconstruction performance for a specific set of principal components. The principal component algorithm provided by the Python package Scikit-learn [Pedregosa et al. 2011] was used to compute the actual components. The first six components are shown in Figure 2. Although it is in general unclear how to attribute specific physical properties to a single component, the first two components show a clear signature of the general oxygen absorption features and the linear model of the surface reflectance.

The uncompressed lookup table contains consists of 2,496,000 spectra, each with a spectral resolution of 4501 channels, which is too large to efficiently compute principal components. Such an amount of data makes the computation of the principal components for the complete dataset numerically challenging. As shown by Hollstein and Lindstrot (2013), this is not necessary, since a randomly selected sub sample of spectra of sufficient size
Table 1. Parameter grid of the lookup table. The table contains 4 \* 5 \* 3 \* 5 \* 2 \* 2 \* 13 \* 8 \* 20 = 2496000 states and corresponding spectra.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( n_p )</th>
<th>( n_{\alpha} )</th>
<th>( n_{\alpha_1} )</th>
<th>( n_{\alpha_2} )</th>
<th>( n_{\mu} )</th>
<th>( n_{\sigma} )</th>
<th>( n_{\varphi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>surface pressure ( \rho )</td>
<td>4</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>aerosol optical thickness ( n_s )</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>aerosol center height ( n_h )</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>aerosol type ( n_t )</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>surface reflectance at 755( nm )</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>surface reflectance at 780( nm )</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>viewing zenith angle ( \mu )</td>
<td>13</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>solar zenith angle ( \mu_S )</td>
<td>8</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>relative azimuth angle ( \phi )</td>
<td>20</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

\[
SNR = \frac{\text{mean}(y)}{\text{std}(y)},
\]

Figure 3 shows the dependency of the mean of the synthetic signal to noise ratio for the whole lookup table with respect to the number of used principal components. The synthetic signal to noise level was defined as:

\[
SNR = \frac{\text{mean}(y_i)}{\text{std}(y_i - \bar{y})},
\]

and where \( y \) is a synthetic measurement and \( \bar{y} \) a reconstructed spectrum and is used as measure a measure of fit quality throughout this paper. The results show clearly, that the reconstruction quality increases strongly with increasing number of principal components. Also, that a number of ten principal components is sufficient to represent the original data with a mean synthetic SNR of approximately 1000.

The results clearly show that the reconstruction quality increases dramatically with increasing number of principal components. The histogram for 15 principal components shows a peculiar dip in the middle of the distribution, which sets this distribution slightly apart from the other ones, but causes no difficulties since the whole distribution shows a much better mean reconstruction quality than when using only five principal components. This effect shows that it is in principle difficult to establish a physical link between a principal component and its effect on the overall reconstruction accuracy when it is added to the reconstruction matrix.

The aerosol optical models were implemented according to [Levy et al. (2007)]. These models are also used by the MODIS aerosol retrieval and were specifically designed to fit observations for different locations on the globe. These models are representative from a MODIS perspective, since they were designed to represent what can be retrieved with MODIS and not for what could be seen in the Oxygen A band. It is beyond the scope of this paper to derive an aerosol model which maximizes the use of all the information in the Oxygen A band, but these models are a good choice nevertheless. They allow to assess which of these widely used models can be distinguished from the discussed type of measurements. From the published optical properties, the urban, neutral, dust, continental, and absorbing types were implemented and Mie calculations using the implementation provided by [Wiscombe (1980)] were used to compute phase functions, extinction cross-sections, and single scattering albedo. Aerosol phase functions at 774.5\( nm \) with respect
to scattering angle and optical thickness are shown in Figure 5.

The approach of this study is to expand on the retrieval error analysis as presented by Sanders and de Haan (2013) and to implement a real-world an actual retrieval scheme, where simulated measurements with realistic random errors are fitted using the fast forward operator. This approach is suited to realistically discuss the difficulties which can arise from multiple minima in the cost function, which can be caused by almost linearly dependent partial derivatives of the forward operator.

The dependency of the partial derivative of the forward operator with respect to aerosol optical thickness and aerosol height is shown in Figure 6 and for surface reflectance and aerosol optical thickness in Figure 7. The partial derivatives were rescaled with a specific decimal power and corresponding multiplied with a scaling factor with appropriate unit, such that both rescaled spectra carry the unit radiance and are of comparable magnitude. By neglecting non-linearity, one can interpret the scaling factors as the specific quantity for both compared parameters, which causes a similar radiance change.

Figure 6 shows clearly that the partial derivatives with respect to aerosol height and aerosol optical thickness are quite variable with respect to functions of the aerosol model and can be largely described by almost linear sections with alternating signs of the slope. Figure 6 shows the partial derivatives with respect regard to aerosol height and aerosol optical thickness, which rather strongly depend on the aerosol model and can be largely described by almost linear sections with alternating signs of the slope. This behavior could potentially lead to multiple minima in the least squares cost function. Also, their differences could help to discriminate between aerosol types. Figure 7 shows that the partial derivatives with respect to aerosol optical thickness and surface reflectivity are strongly linearly correlated, with a strong dependency on aerosol type. This indicates one of the main theoretical problems with deriving the aerosol height from the Oxygen A band alone. Crucial for the retrieval accuracy will be the proper discrimination of surface reflectivity and aerosol optical depth. Both parameters are highly variable in space and time on short scales, such that prior knowledge with sufficiently small uncertainty will likely be unavailable.
4 Synthetic Measurements

Scope of this paper is the simulation of general hyperspectral radiometers with arbitrary spectral resolution, spectral sampling, and signal to noise level. As generic noise model, a random noise vector \( n \) was added to a forward operator simulation to produce a synthetic measurement \( y_{\text{in}} \) for the Oxygen A band:

\[
y_{\text{in}} = y + n(y), \quad n(y) = r_{-1,1} \text{mean}(y)/\text{SNR},
\]

where \( r_{-1,1} \) is a vector with the dimension of the simulations which contains random numbers between \(-1\) and \(1\), \( \text{SNR} \) is the prescribed signal to noise ratio, and \( \text{mean}(y) \) denoted the spectral average for the complete Oxygen A band. With this definition, the synthetic SNR of a spectral fit is approximately the prescribed SNR number.

A more realistic model can be used with a specific type of instrument in mind, but would at the same time be less general for other types of instruments. This makes the setup of a generic noise model for this type of study difficult. Although the chosen model is very simple and generic, it is sufficiently close to a realistic noise model (e.g., see Aiazzi et al., 2006) and avoids the problem of almost zero relative error for small measurements when using pure multiplicative noise. In addition, the model is controlled by a single parameter which makes the discussion of the noise more coherent.

5 Inversion Scheme

The fast forward operator was implemented in FORTRAN, parallelized using OpenMP, and run on a standard desktop computer with 8GB of main memory and an Intel i7-3770 CPU with 4 cores running at 3.20GHz. The linear interpolation - multi-linear interpolation in all dimensions of the lookup table was implemented in FORTRAN such that the Jacobian of the interpolated function is computed analytically without much computational overhead. In a way, that the first order derivatives, which we will call Jacobians throughout this paper, are computed analytically from the interpolation formula. This approach is numerically stable and needs only little extra computational time.

Results for a simple benchmark are shown in Table 2. Each benchmark result is based on the computation of 50,000 randomly selected states. The benchmark shows that the run time of the forward operator is almost independent from spectral resolution and sublinearly increases with increasing number of principal components.

The Levenberg Marquardt optimization routine \textit{lmdner} from the MINPACK project provided by More et al. (1984) was used to minimize the sum of least squares between simulated measurements and results of the fast forward operator. As discussed in Section 3 no prior knowledge is assumed since for the crucial variables aerosol type, aerosol height, aerosol optical thickness, and surface reflectance, prior knowledge will likely not be available with sufficiently small error.

The \textit{original lmdner} routine is implemented for an unbounded problem - To and was modified to ensure that the search range - minimization search stays within the bounds of the lookup table. This is achieved by modifying a computed next iteration step was modified such that the bounds of the lookup table can only be reached asymp-
tatively. The modification replaces the computed stepsize by half of the distance between the actual position and the boundary. A certain minimum step size for aerosol optical depth was chosen as a convergence criterion for the iteration.

6 Noise Model

Scope of this paper is the simulation of general hyperspectral radiometers with arbitrary spectral resolution, spectral sampling, and signal to noise level. As generic noise model, a random noise vector $n$ was added to a forward operator simulation to produce a synthetic measurement $y_m$:

$$y_m = y + n(y), \quad n(y) = r_{-1,1} \text{mean}(y)/\text{SNR},$$

where $r_{-1,1}$ is a vector with the dimension of the simulations which contains random numbers between $-1$ and $1$ and $\text{SNR}$ is the prescribed signal to noise ratio. With this definition, the synthetic SNR of a spectral fit is approximately the prescribed SNR number.

Although this is a very simple noise model, it is sufficiently close to a realistic noise model (e.g., see Aiazzi et al., 2006) and avoids the problem of almost zero relative error for small measurements when using pure multiplicative noise. The lmder routine is called 20 times with randomly selected starting values for each inversion attempt and the best result in terms of cost function value is returned as final result. This approach eliminates effects caused by possible multiple minima in the cost function, or a sensitivity of the inversion for the starting values.

6 Synthetic Retrieval Results

The sensitivity study is based on a Monte Carlo approach. A number of 50,000 retrievals with randomly selected state vectors is performed for a given setting of spectral resolution, spectral sampling, signal to noise ratio, prior aerosol type information, and surface inhomogeneity. The complete parameter space of the lookup table as given in Table I was used for the generation of random atmospheric states. Surface inhomogeneity is modeled by applying the independent column approach, where the mean value over $N_o = 9$ simulations using a range of different surface reflectances is taken. The resulting set of prescribed state vectors and inversion results is then analyzed by means of scatter plots and mean values.

Two named cases are distinguished throughout the analysis, the best case and the realistic case scenario. The best case scenario is characterized by a retrieval with known aerosol type and simulations with homogeneous surface reflectance, while for the realistic case the aerosol type is a free inversion parameter and surface inhomogeneity is taken into account. The two cases can be used to study the effect of an unknown aerosol type, which is believed to have only minor effects (compare Sanders and de Haan, 2013), and also the effect of the finite spatial resolution of a hypothetical sensor. In general, spectral resolution, spectral sampling, and spatial resolution are competing factors for an instrument with given external constraints. Increasing the spectral resolution decreases the amount of energy within each channel, which can be compensated with more sensitive detectors, longer integration time, or a larger footprint on the surface of the Earth. The realistic case is therefore more realistic for hyperspectral instruments such as TROPOMI or TANSO-FTS, where an instrument with lower spectral resolution such as FLEX offers much higher spatial resolution.

Spectral resolution and sampling are two free and independent parameters of the fast forward operator setup and their effects can be studied independently. Here, we focus our analysis to the retrieval of aerosol parameters and simplify the parameter space to cases where spectral resolution and sampling are equal. This implies that no resampling is considered, which can help to reduce the impact of random noise for real instruments.

It is of great importance for the general applicability of the inversion results that the scheme robustly finds the global minimum of the cost function. This is ensured by a large number of random starting values and a comparison of the resulting synthetic SNR value from the fit residual with the prescribed one. If the achieved residual is in the order of the prescribed noise, the inversion is successful and different schemes might only be more efficient in the needed computational burden or memory use. Figure 8 shows the mean inverse signal to noise ratio of the fit residual with respect to spectral resolution, prescribed signal to noise ratio, and the best case and realistic case scenario. The results show clearly that the inversion succeeds and is capable of finding a minimum in the cost function which can be completely explained by noise. This is also highlighted in Figure 1 where several simulated spectra, inversion results, and resulting residuals are shown for three spectral resolutions. The general inversion residual is within the prescribed noise, which indicates that an improvement above the results presented here is only possible by introducing further measurements or additional prior knowledge. However, such an analysis is beyond the scope of this paper.

As discussed above, the methodology of this study is purely Monte Carlo like and an overview about results for the best case, realistic case, and synthetic SNR value of 500, and varying spectral resolution is shown in Figure 9. The top row of subfigures shows the effect of unknown aerosol type and surface inhomogeneity for constant signal to noise ratio, spectral resolution, and spectral sampling. While the results for the aerosol height retrieval even slightly improve,
the retrieval of aerosol optical depth is affected by relaxing these constraints. The aerosol height retrieval is almost unaffected, its bias is slightly reduced and the slope is closer to one. The scatter for aerosol optical depth retrieval increases and the slope deviates more from one. In this respect, the aerosol height retrieval is more stable with respect to aerosol type and surface inhomogeneity than the retrieval of aerosol optical depth. The bottom row of the figure shows the effect of decreasing the spectral resolution and spectral sampling to 0.1nm. As for the case with higher spectral resolution and sampling, the retrieval of aerosol height is more stable, while the retrieval of aerosol optical depth is more strongly affected.

The presented scatter plots can be seen as a best estimate for a real world validation of a retrieval scheme. The validation of the retrieval scheme in reality. Its success can be measured in terms of absolute accuracy, but also relative to given user requirements, which in general strongly depend on a specific application. Aim of this paper is to be rather general and not to base the discussion on a certain user requirement and application. Hence, throughout this paper the absolute retrieval accuracy is taken as measure.

Visible in all scatter plots are minor artifacts which are caused by the grid points of the tabulated aerosol optical depth and aerosol height. The artifacts are horizontal lines of increased occurrence for a parameter value which is a grid point in the lookup table (compare also with Table 1). A real-world retrieval could simply avoid these artifacts by using a finer grid in the lookup table, which would then better represent the nonlinear response of the simulations with respect to these parameters. These artifacts are shown in Figure 9 and 10 to highlight the effects of a finite resolution in the lookup table, but are excluded in the further analysis.

Similar scatter plots, but for a prescribed signal to noise ratio of 250 are shown in Figure 10. The general distribution of scatter points is much wider than in Figure 9, although the regression lines are only slightly affected. It will depend on the desired application whether such scatter can be accepted.

In contrast to other shown inversion results, each subfigure of Figure 9 and 10 is based on the inversion of 10,000 randomly selected state vectors. Its computation on a standard PC with Intel i7-3770 CPU took approximately 30min (see also the benchmark results in Section 5). Each inversion for a single spectrum was repeated 20 times with randomly selected starting values to ensure convergence to the global minimum of the least squares cost function.

The discussed results provide evidence, that the retrieval of aerosol height will depend on instrumental parameters like spectral resolution, spectral sampling, and signal to noise ratio, but also on the scene itself. This is included in the discussion by prescribing or removing the aerosol type information and introducing surface inhomogeneities. Results regarding this assumption are shown in Figure 11, which compiles the inversion success for surface pressure, aerosol optical thickness, and aerosol height. Inversion success was defined as the mean absolute radiance residual for the 90% best cases in rescaled units as they were used in Figure 9 and 10. Zero indicates a perfect mean inversion while one indicates that the mean residual is in the order of the maximum of the range of the retrieved quantity. Surface pressure is shown merely as reference, its retrieval success depends almost only on signal to noise error and decreases only slightly with increasing spectral resolution.

The results for aerosol optical depth and aerosol height behave quite differently. While being sensitive to spectral resolution, the inversion success strongly depends on the signal to noise level. The strongest increase for aerosol optical thickness can be seen when decreasing the spectral resolution from 1nm to 0.1nm. From that on, only minor improvements in the retrieval can be achieved by increasing the spectral resolution of the instrument. Again, it will depend on user requirements whether a possibly small gain in retrieval accuracy from increasing the spectral resolution is feasible. Similar, a similar, but weaker behavior can be seen for the retrieval of aerosol height. Depending on the signal to noise level, the retrieval of aerosol height could become worse for increased spectral resolution. It should be noted, that these results describe the inversion success with respect to spectral resolution at constant signal to noise range. When increasing the spectral resolution, one automatically increases the dynamic range of the signal since the fine oxygen absorption lines become better and better resolved. While these features carry information about the aerosol height, they are strongly affected by noise, which can lead to a decrease in retrieval accuracy. For a real instrument, increasing the spectral reso-
The overall effect is minor and is more pronounced for the aerosol optical thickness. A conclusion is that the retrieval of aerosol height is robust against variations in aerosol type and surface homogeneity. This conclusion is valid for the total physical space which has been discussed here. It could be exploited in much more detail, e.g., by analyzing it with respect to specific aerosol optical parameters and surface conditions, but is left as subject for future research.

Although beyond the scope of this paper and likely of any aerosol retrieval based purely on the Oxygen A band, the presented scheme allows to investigate to what extent possible...
Conclusions

A fast forward operator for the simulation of hyperspectral radiances in the Oxygen A band was described, benchmarked on a standard computer, and applied for a sensitivity study concerning the retrieval of aerosol optical depth and aerosol height. Study parameters were spectral resolution, spectral sampling, signal to noise ratio, aerosol type information, and surface inhomogeneity. The study showed that the retrieval generally benefits from higher spectral resolution, with the strongest increase in retrieval accuracy above a spectral resolution of 0.1 nm. Signal to noise ratio strongly affects the retrieval and is a key parameter when designing an instrument and a retrieval scheme. The retrieval of aerosol height seems to be robust even when aerosol type information is missing and when surface inhomogeneity is introduced. These conclusions hold for the retrieval of a single aerosol layer when a single aerosol layer is present. More complex vertical profiles should be subject to future research. Evidence was found that the Oxygen A band contains valuable information about the aerosol type, which could be used in an aerosol retrieval which utilizes additional spectral bands. The dependency with respect to the temperature profile was not discussed here and might be subject to future research.

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References


ESA: GMES SENTINELS 4 AND 5 MISSION REQUIREMENTS TRACEABILITY DOCUMENT, issue 1, revision 0, Tech. rep., European Space Research and Technology Centre, available at: http://esamultimedia.esa.int/docs/EarthObservation/S4_5p_MRTD_issue1_0 authorised.pdf (last access 22 October 2013), 2012.


6-8339-2013 http://www.atmos-meas-tech-discuss.net/6/8339/2013


