Effect of sampling variation on error of rainfall variables measured by optical disdrometer

X. C. Liu, T. C. Gao, and L. Liu

College of Meteorology and Oceanography, PLA University of Science and Technology, Nanjing, China

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Correspondence to: T. C. Gao (2009gaotc@gmail.com)

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Abstract

During the sampling process of precipitation particles by optical disdrometers, the randomness of particles and sampling variability has great impact on the accuracy of precipitation variables. Based on a marked point model of raindrop size distribution, the effect of sampling variation on drop size distribution and velocity distribution measurement using optical disdrometers are analyzed by Monte Carlo simulation. The results show that the samples number, rain rate, drop size distribution, and sampling size have different influences on the accuracy of rainfall variables. The relative errors of rainfall variables caused by sampling variation in a descending order as: water concentration, mean diameter, mass weighed mean diameter, mean volume diameter, radar reflectivity factor, and number density, which are independent with samples number basically; the relative error of rain variables are positively correlated with the margin probability, which is also positively correlated with the rain rate and the mean diameter of raindrops; the sampling size is one of the main factors that influence the margin probability, with the decreasing of sampling area, especially the decreasing of short side of sample size, the probability of margin raindrops is getting greater, hence the error of rain variables are getting greater, and the variables of median size raindrops have the maximum error. To ensure the relative error of rainfall variables measured by optical disdrometer less than 1 %, the width of light beam should be at least 40 mm.

1 Introduction

Precipitation is defined as the liquid or solid products of the condensation of water vapor falling from clouds or deposited from air onto the ground (WMO, 2008), the measurement of precipitation is the focus of research in the fields such as meteorology, hydrology and environment. There are many instruments available to measure precipitation, according to the principle, these instruments could generally be divided into two types as follows (Joss and Waldvogel, 1969): (1) time integrating devices and
(2) volume integrating instruments. The first type of instrument measures the number and diameters of particles that reach a sampling surface during a given period of time, the disdrometer RD-69 (Sheppard and Joe, 1993), the Optical Spectro-Pluviometer (Salles et al., 1998), OTT PARSIVEL disdrometer (Battaglia et al., 2009), the 2-D Video Disdrometer (Kruger and Krajewski, 2002), and Linear Array Optical Precipitation Sensor (LAOPS) (Gao et al., 2012, 2013) work in this way. The other type of instrument measures the number and diameters of particles in a given sampling volume at one moment, the raindrop camera (Saylor et al., 2002), and the vertically pointing doppler radar (Peters et al., 2002) all work based on this principle. These instruments have been widely used at home and abroad, which have played an important role in disclosing the feature of precipitation, evaluation of rainfall enhancement, and etc.

Although the comparative observations with the disdrometers and rain gauge verify the accuracy of rain rate measured by disdrometers, the accuracy of the drop size distribution, number density, and other micro-physical variables of precipitation can not be estimated, the reason is that there's no standard instrument that can obtain the true values, nor standard environment that can simulate the precipitation. Various comparative observations with multiple disdrometers show that there are obvious differences between raindrops size distribution obtained by different disdrometers (Tokay et al., 2001, 2003; Lohnert et al., 2011), which makes it difficult to the effective application of micro-physical data of precipitation.

In general, this variability can be attributed to three factors as follows (Uijlenhoet et al., 2006): (1) climatological factors, rainfalls of different types have different properties, (2) physical factors, meteorological conditions vary during rainfall events, and (3) instrumental factors, related to the instruments and their measurement principles. The former two are the main contents of climatic analysis, meteorological research, and weather service, while the latter includes sampling variation, sensor sensitivity, and instrument malfunctioning, where the sampling variation caused by sample size is an important factor influencing the accuracy of precipitation measurements. Because of the randomness associated with sampling from a population of raindrops, sampling
variability can cause the variations of water concentration, reflectivity factor, maximum particles size (Smith et al., 1993), the noise-type error of the sampling can cause discrepancies greater than 30% in the number of hailstones on the hailpad (Wirth et al., 1983), therefore a large sample is necessary to get a good estimate of rain intensity $R$ and radar reflectivity factor $Z$ (Joss and Waldvogel, 1969). The effect of sampling variation on raingroup size measurements in stationary rainfall by using a stochastic model of the microstructure of rainfall was researched by Uijlenhoet et al. (2006), and the sampling effects in DSD measurements in non-stationary rain was simulated by Berne (Berne and Uijlenhoet, 2005). But the accuracy of drop size distribution and velocity distribution measured by various optical disdrometers have not been evaluated thoroughly.

In order to quantify the effect of sampling variation on the error of rainfall measurement using optical disdrometer, we begin with the marked point process model of raingroup size distribution, aiming at the existing PMS GBPP-100 disdrometer, OTT PARSIVEL disdrometer, 2DVD, LAOPS, and etc, the effect of sampling variation on measurement of drop size distribution and velocity distribution are analyzed by Monte Carlo simulation. The conclusions of this paper can provide a theoretical reference for the promotion of the accuracy of precipitation DSD measurement, improvement of disdrometer, and better application of DSD data.

2 Theory and methodology

2.1 Drop size distribution

Generally, the power-law parameters for MP distribution (Marshall and Palmer, 1948) are widely used for describing precipitation, but it is only accurate for stable rainfall from stratiform clouds (Joss and Gori, 1987; Carbone and Nelson, 1978; Willis, 1984). It is now widely accepted that the drop size distribution of natural rain is better represented by a gamma distribution (Ulbrich, 1983; Testud et al., 2001)
\[ N(D) = N_0 D^\mu e^{-\lambda D} \]  

where \( N(D) \) is the number concentration of drops of equivalent diameter \( D \), \( N_0 \) is the total drop concentration, \( \mu \) is the order of the gamma size distribution, and \( \lambda \) is the slope of the gamma size distribution. These parameters have different values for different rainfall cases. Three representative types of rain (stratiform rain, mixed stratiform and cumulonimbus rain, cumulonimbus rain) are obtained by statistic analysis of drop size distribution of different rain from Jiangsu (Pu et al., 2010), He’nan (Shi et al., 2004), Heilongjiang (Yuan et al., 2001), Liaoning (Chen et al., 1998), Ningxia (Niu et al., 2002), and Guangdong (Li et al., 2010) of China, the parameters of DSD are summarized in Table 1.

Figure 1 shows the drop size distribution of three types of rainfall \((R = 5 \text{ mm h}^{-1})\), it can be found that stratiform rain has the most small raindrops and the least large raindrops, the cumulonimbus rain has the least small raindrops and the most large raindrops, while the mixed stratiform and cumulonimbus rain is between the former two.

2.2 Marked point process model

Based on the drop size distribution of rainfall above, marked point process model (Smith, 1993) is adopted and modified, the sampling processes of rainfall by various disdrometers with different sampling sizes are simulated in this paper. The marked point process can be divided into two categories: (1) the point process represents the drops' positions in the sampling volume, and (2) the mark process represents the drops' diameters. In this paper, the positions of drops are assumed to be randomly distributed in space (called homogeneous Poisson model), which is supported both by theoretical and by empirical evidence (Sasyo, 1965; Joss and Waldvogel, 1969); the diameter and number of raindrops yields to the drop size distribution function \( N(D) \). According to the homogeneous Poisson model, the number density of drops per unit volume is \( N_T \), the number of drops in a sampling volume \( V_S \) is:
\[ n_s = V_S N_T \] (2)

The drop size distribution can be characterized by a probability density function \( p(D) \), \( p(D)dD \) is the probability that the drop in the interval \([D \sim D + dD]\) (Uijlenhoet et al., 2006). The drop size distribution \( N(D) \) is a function of \( N_T \) and \( p(D) \):

\[ N(D) = N_T p(D) \] (3)

Assuming that the terminal velocity of raindrops in the air depends exclusively on their diameter, and drops do not interact with each other, the time interval of raindrops reach the sampling surface are distributed exponentially. Thus, the probability density function \( \phi(t) \) of the interval between two consecutive drops that arrival at the sampling surface is

\[ \phi(t) = \lambda S e^{-\lambda St} \] (4)

where \( S \) is the sampling surface, \( \lambda \) is the arrival rate, which denotes the number of drops arriving at the sampling surface per unit area and per unit time, it can be calculated by the drop size distribution \( N(D) \) and the drop terminal fall velocity \( \nu(D) \):

\[ \lambda = \int \nu(D) N(D) dD = N_T \langle \nu \rangle \] (5)

where \( \langle \nu \rangle \) is the average terminal velocity of raindrops in the sampling volume:

\[ \langle \nu \rangle = \int \nu(D) p(D) dD \] (6)

The terminal velocity of raindrops \( \nu \) can be calculated according to the equivalent diameter \( D_{eq} \):

\[
\nu = \begin{cases} 
30.75D_{eq}^2 & D_{eq} < 0.1 \text{mm} \\
3.8D_{eq} & 0.1 \text{mm} < D_{eq} < 1 \text{mm} \\
4.21 \sqrt{D_{eq}} & D_{eq} > 1 \text{mm} 
\end{cases}
\] (7)
The number of drops \( n_s \) that reach the surface \( S \) during the period \( t \) is

\[ n_s = \lambda S t \]  

(8)

The probability density of the drops arriving at the sampling surface is denoted by \( p_A(D) \), which differs from \( p(D) \), the relation between them is

\[ p_A(D) = \frac{\nu(D)}{\langle \nu \rangle} p(D) \]  

(9)

Then the drop size distribution \( N_A(D) \) becomes

\[ N_A(D) = \lambda p_A(D) \]  

(10)

Such that \( N_A(D)dD \) represents the number of raindrops with diameters between \( D \sim D+dD \) that arrive at a surface per unit area and per unit time. In principle, \( N(D) \) can be used when analyzing processes involving a sample volume (such as radar reflectivity factor, liquid water concentration), and \( N_A(D) \) can be used when analyzing processes related to fluxes (such as rain rate).

### 2.3 Calculation of rainfall variables

The parameters characterizing rainfall include several diameter variables and volumetric variables, such as number density, rain rate, water concentration, radar reflectivity factor. In this paper, these variables can be calculated as follows.

1. Mean diameter:

\[ D_m = \frac{\sum N(D)D}{\sum N(D)} \]  

(11)
(2) Volume mean diameter:
\[ D_{vm} = \left[ \frac{\sum N(D)D^3}{\sum N(D)} \right]^{1/3} \]  
(12)

(3) Mass-weighted mean diameter:
\[ D_{mm} = \frac{\sum N(D)D^4}{\sum N(D)D^3} \]  
(13)

Volumetric variables \( Q \) can be calculated in terms of \( N(D) \)
\[ Q_p = \int_0^\infty Q(D)N(D)dD = \sum Q(D)N(D) \]  
(14)

where (1) Number density \( (N_d) \): \( Q(D) = 1 \) (0-order moment); (2) water concentration \( (W) \): \( Q(D) = \pi \rho D_{eq}^3/6 \) (3rd-order moment), \( \rho \) is the density of water, \( D_{eq} \) is the equivalent diameter of raindrop; (3) radar reflectivity factor \( (Z) \): \( Q(D) = D_{eq}^6 \) (6th-order moment).

Because the optical disdrometer can only measure the horizontal dimension of raindrops, the vertical dimension of raindrops can only be calculated by the empirical nonspherical shape of raindrops (Beard et al., 2010). To simplify the calculation, the axis ratio of width and height of raindrops is adopted to describe the shape of raindrops.

\[ b/a = \begin{cases} 
1 & D_{eq} \leq 1 \text{ mm} \\
1.075 - 0.075D_{eq} & 1 \text{ mm} < D_{eq} < 5 \text{ mm} \\
0.7 & D_{eq} \geq 5 \text{ mm}
\end{cases} \]  
(15)
3 Simulation results of drop size distribution

Considering the OTT PARSIVEL disdrometer is the most widely used instrument measuring precipitation, the PARSIVEL measuring rational is used to retrieve the drop size distribution and velocity distribution with the simulation of sampling process of raindrops. The sample area of OTT PARSIVEL is 180 mm × 27 mm, according to the retrieval rational of OTT PARSIVEL, the raindrops fall at the margin of sampling light beam would be detected and eliminated, then the sampling area would be 180 mm × (27 mm − 2D_{eq}^{par}), where D_{eq}^{par} is the equivalent diameter of margin raindrops, while there’s no similar method used by other instruments.

The effect of sample number, rainfall intensity, and rainfall types on the measurement of drop size distribution are analyzed, and then the different sampling volume (PMS GBPP-100 Disdrometer (GBPP-100), OTT PARSIVEL Disdrometer (PARSIVEL), Laser Precipitation Monitor (LPM), Optical Spectro-Pluviometer (OSP), 2-D Video Disdrometer (2DVD), Linear Array Optical Precipitation Sensor (LAOPS)) on the measurement of drop size distribution is discussed in this section.

3.1 Effect of samples number on DSD

Sampling process of raindrops from stratiform rainfall are simulated by using marked point process model in this section, the total samples number is set to 116, 585, 3509, 11693, respectively. Because one single simulation shows obvious randomness, 10 runs for each simulation are necessary, and then the mean values of rainfall variables and their relative error can be calculated.

Table 2 lists the mean diameter, volume mean diameter, mass-weighted mean diameter, number density, water concentration, and radar reflectivity factor of rainfall simulation of different samples number. In general, the probability of raindrops (stratiform rainfall, R = 5 mm h⁻¹) fall at the margin of sampling volume is about 3.8%; and the rainfall variables vary little with the samples number. The mean diameter of raindrops is 0.98 mm, the volume mean diameter of raindrops is 1.2 mm, the mass-weighted mean
diameter of raindrops is 1.6 mm, the number density of raindrops is about \(2.6 \times 10^3\) m\(^{-3}\), the water concentration is about 2.4 g m\(^{-3}\), and the radar reflectivity factor is about 100 dBZ.

Figure 2 shows the variation of the relative error of rainfall variables with the samples number. It can be found that the relative error is relative large when the samples number is less than 200 because of great randomness. When the samples number exceed 500, the relative error of rainfall variables vary little with the samples number, the error of which in a descending order as: mean diameter, water concentration, volume mean diameter, mass-weighed mean diameter, number density, and radar reflectivity factor, their values are \(-4.49\%\), \(-3.38\%\), \(-2.11\%\), \(-0.85\%\), \(-0.69\%\), and \(-0.56\%\).

### 3.2 Effect of rainfall intensity on DSD

Table 3 lists the stratiform rainfall variables of different rainfall intensity, it can be found that all of the rainfall variables increase with the increasing of rainfall intensity. The margin number of total samples number increase with the increasing of rainfall intensity, the reason is that the probability of raindrops fall at the margin of sampling volume increases with the increasing of large raindrops samples, which is positively correlated with the rain rate, it can be validated by Fig. 3. In general, the small drops of retrieved DSD is more than that of simulated DSD, while the large drops of retrieved DSD is less than that of simulated DSD.

Figure 4 shows the variation of the relative error of stratiform rainfall variables with the rainrate, the relative error of rainfall variables increases obviously with the increasing of rainrate. The relative error of mean diameter, volume mean diameter, mass-weighed mean diameter, radar reflectivity factor are less than 0, indicating that these four variables would be underestimated by optical disdrometers; the relative error of number density is less than 0 when \(R < 8\) mm h\(^{-1}\), and greater than 0 when \(R > 8\) mm h\(^{-1}\), indicating that number density would be underestimated when \(R < 8\) mm h\(^{-1}\) and overestimated when \(R > 8\) mm h\(^{-1}\); the relative error of water concentration is less than 0
when $R < 36 \text{ mm h}^{-1}$, and greater than 0 when $R > 36 \text{ mm h}^{-1}$, indicating that water concentration would be underestimated when $R < 36 \text{ mm h}^{-1}$ and overestimated when $R > 36 \text{ mm h}^{-1}$. The relative error of number density has the maximum variation with rain rate, the reason is that the mean diameter increase and effective sampling area decreases with the increasing of rainrate; while the relative error of radar reflectivity factor has the minimum variation with rain rate, mainly because of a logarithmic transformation of large accumulated error caused by a big probability of margin fallers.

### 3.3 Effect of rainfall type on DSD

Table 4 lists the rainfall variables of different DSD ($R = 5 \text{ mm h}^{-1}$). There are a largest number of raindrops in the stratiform rain and a smallest number of raindrops in the cumulonimbus rain, while the number of raindrops in the mixed stratiform and cumulonimbus rain is between the above two types, while the $D_m$, $D_{vm}$, $D_{mm}$, $W$, and $Z$ of rainfalls of three types are just the opposite. The probabilities of margin raindrops from different rainfalls are in consonance with the number density of large raindrops. The probability of margin raindrops in different type’s rainfalls in a descending order is: cumulonimbus rain, mixed stratiform and cumulonimbus rain, and stratiform rain.

The comparisons of the simulated DSD and retrieved DSD in Fig. 5 show that the small drops of retrieved DSD is more than that of simulated DSD, while the large drops of retrieved DSD is less than that of simulated DSD, and the DSD of three types rainfalls present above consistent characteristics. The simulated DSD of cumulonimbus rain has the largest error, especially for large raindrops because of the large random error caused by the large probability of raindrops falling at the margin, which associated with the features of different DSD.

Table 5 lists relative error of rainfall variables from different DSD ($R = 5 \text{ mm h}^{-1}$), the largest error occurred in the measurement of cumulonimbus rain, the smallest error occurred in the measurement of stratiform rain. The relative error of cumulonimbus rainfall variables in a decreasing order are $N_d$, $W$, $D_m$, $D_{vm}$, $D_{mm}$, and $Z$; the relative
error of mixed stratiform and cumulonimbus rainfall variables in a decreasing order are $D_m$, $D_{vm}$, $N_d$, $W$, $D_{mm}$, and $Z$; the relative error of stratiform rainfall variables in a decreasing order are $D_m$, $W$, $D_{vm}$, $D_{mm}$, $Z$, and $N_d$.

### 3.4 Effect of sampling size on DSD

There are several optical disdrometers with different sampling sizes at present: PMS GBPP-100 Disdrometer (630mm × 12.6mm) (Fraile et al., 2009), OTT PARSIVEL Disdrometer (180mm × 27mm) (Battaglia et al., 2009), Laser Precipitation Monitor (225mm × 20mm) (King et al., 2010), Optical Spectro-Pluviometer (250mm × 40mm). (Salles et al., 1998), and Linear Array Optical Precipitation Sensor (300mm × 40mm). (Gao et al., 2012) have a single light beam, while 2-D Video Disdrometer (100mm × 100mm) (Kruger and Krajewski, 2002) has two light beams. Considering the representativeness of different sampling area, effect of sampling size on the error of rainfall variables is discussed aiming at these six instruments. By using the numerical simulation of sampling process, the relative error of stratiform rainfall variables ($R = 10 \text{ mm h}^{-1}$) can be obtained in Table 6.

It can be seen that the GBPP-100 has the smallest width of light beam, the maximum probability of margin raindrops, and the greatest error of rainfall variables; 2DVD has the largest width of light beam, the minimum probability of margin raindrops, and the least error of rainfall variables. It can be concluded that the width of light beam is the main factor that influence the probability of margin raindrops, for the same width of sampling volume, the relative errors of rainfall variables are negatively correlated with the length of sampling volume. The width of sampling size should be no less than 40 mm to make sure the relative error of each rainfall variable less than 1%, the OSP, LAOPS and 2DVD can meet this criterion. In general, the water concentration retrieved has the largest error; the radar reflectivity factor retrieved has the small error. The number density measured by OTT PARSIVEL has a larger relative error, because the margin raindrops were detected and eliminated by two additional photo diodes. (Battaglia et al., 2009).
4 Simulation results of drop velocity distribution

Optical disdrometer with a single light beam can measure the fall velocity of particles from the particle size (by assuming a fixed relationship between horizontal and vertical dimensions) and the time duration of light signals. When the raindrops fall at the margin of light beam, the error of measured horizontal dimensions might induce a certain error to the drop velocity distribution. Using the similar simulation method in Sect. 2, the effect of sampling variation on the drop velocity distribution is discussed as follows. The velocity of raindrops yields the equation (Laws, 1941; Gunn and Kinzer, 1949):

$$
V = \begin{cases} 
30.4D_{eq}^2 & D_{eq} < 0.05 \text{mm} \\
1.91 \left[ 1 - \exp\left( -\frac{D_{eq}}{0.316} \right)^{1.754} \right] & 0.05 \text{mm} \leq D_{eq} \leq 0.3 \text{mm} \\
9.32 \left[ 1 - \exp\left( -\frac{D_{eq}}{1.77} \right)^{1.147} \right] & D_{eq} > 0.3 \text{mm}
\end{cases}
$$

(16)

Figure 6 shows the drop velocity distributions of stratiform rainfall measured by GBPP-100, LPM, OPS, and LAOPS respectively, the probability of margin raindrops is 8.6 %, 5.6 %, 3.5 %, and 3.2 %, which are in accordance with the sampling size of different instruments. It can be found that the error of velocity distribution increases with the increasing of probability of margin raindrops, and the velocity of median size raindrops (0.5 mm $\leq D_{eq} \leq$ 3 mm) have the maximum error due to the large number of median size raindrops fall at the margin of light beam, there is few small ($D_{eq} < 0.5$ mm) and large ($D_{eq} > 3$ mm) raindrops fall at the margin of light beam, the probable reason is that the small raindrops fall at the margin of light beam have a small probability; the large raindrops fall at the margin of light beam have a large probability, but the number density of large raindrops is extremely low.

Figure 7 shows the drop velocity distribution of stratiform rain, mixed stratiform and cumulonimbus rain, and cumulonimbus rain measured by LAOPS, the probability of margin raindrops of three DSDs is 2.7 %, 3.4 %, and 3.9 %. It can be found that the...
error of drop velocity distribution increases with the increasing of probability of margin raindrops, which is consistence with that of DSD. The error of drop velocity distribution from different rainfalls in a descending order as: cumulonimbus rain, mixed stratiform and cumulonimbus rain, and stratiform rain, in which the drop velocity distribution of median size raindrops have the maximum error, $0.5 \text{ mm} \leq D_{eq} \leq 2 \text{ mm}$ for stratiform rain, $0.5 \text{ mm} \leq D_{eq} \leq 3 \text{ mm}$ for mixed stratiform and cumulonimbus rain, $1 \text{ mm} \leq D_{eq} \leq 4 \text{ mm}$ for cumulonimbus rain.

5 Conclusions

Based on the marked point process model of raindrop size distribution, the quantitative effect of sampling variation on drop size distribution and drop velocity distribution measured by PMS GBPP-100 Disdrometer, OTT PARSIVEL Disdrometer, Laser Precipitation Monitor, Optical Spectro-Pluviometer, 2-D Video Disdrometer, and Linear Array Optical Precipitation Sensor are analyzed by Monte Carlo simulation. The results show that the samples number, rain rate, rainfall type, and sampling size have different influences on the accuracy of rainfall variables. The errors of rainfall variables caused by sampling variation are independent with samples number; for the same rain rate and rainfall type, the relative error of rainfall variables in a descending order as: water concentration, mean diameter, mass weighed mean diameter, mean volume diameter, radar reflectivity factor, and number density; the relative error of rain variables are positively correlated with the margin probability, which is also positively correlated with the rain rate and the mean diameter of raindrops. The sampling size is one of the main factors that influence the margin probability, with the decreasing of sampling area, especially the decreasing of short side of sample size, the probability of margin raindrops is getting greater, hence the error of rain variables are getting greater, and the variables of median size raindrops have the maximum error. It should be noted that to ensure the relative error of rainfall variables measured by optical disdrometer less than 1 %, the width of light beam should be at least 40 mm.
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References


### Table 1. Parameters of raindrop size distribution of different types.

<table>
<thead>
<tr>
<th>Type of precipitation</th>
<th>$N_0(R), (\text{m}^{-3} \text{mm}^{-1})$</th>
<th>$\lambda(R), (\text{mm}^{-1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stratiform Rain</td>
<td>$2.1 \times 10^4 R^{-0.384}$</td>
<td>$5.38 \times R^{-0.186}$</td>
</tr>
<tr>
<td>Mixed Stratiform and Cumulonimbus Rain</td>
<td>$2.3 \times 10^3 R^{-0.384}$</td>
<td>$3.42 \times R^{-0.186}$</td>
</tr>
<tr>
<td>Cumulonimbus Rain</td>
<td>$0.4 \times 10^3 R^{-0.384}$</td>
<td>$1.74 \times R^{-0.186}$</td>
</tr>
</tbody>
</table>
Table 2. Stratiform rainfall variables of different samples number ($R = 5 \text{ mm h}^{-1}$).

<table>
<thead>
<tr>
<th>Samples number</th>
<th>Margin probability</th>
<th>$D_m$ (mm)</th>
<th>$D_{vm}$ (mm)</th>
<th>$D_{mm}$ (mm)</th>
<th>$N_d$ (m$^{-3}$)</th>
<th>$W$ (g m$^{-3}$)</th>
<th>$Z$ (dBZ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>116</td>
<td>4.3 %</td>
<td>0.97</td>
<td>1.17</td>
<td>1.55</td>
<td>2652</td>
<td>2.21</td>
<td>98.89</td>
</tr>
<tr>
<td>585</td>
<td>3.8 %</td>
<td>0.98</td>
<td>1.20</td>
<td>1.67</td>
<td>2675</td>
<td>2.44</td>
<td>102.97</td>
</tr>
<tr>
<td>3509</td>
<td>3.6 %</td>
<td>0.98</td>
<td>1.21</td>
<td>1.71</td>
<td>2674</td>
<td>2.50</td>
<td>104.13</td>
</tr>
<tr>
<td>11693</td>
<td>3.8 %</td>
<td>0.99</td>
<td>1.22</td>
<td>1.72</td>
<td>2673</td>
<td>2.52</td>
<td>104.77</td>
</tr>
</tbody>
</table>
Table 3. Stratiform rainfall variables of different rainrate.

<table>
<thead>
<tr>
<th>Rain rate (mm h(^{-1}))</th>
<th>Samples number</th>
<th>Margin probability</th>
<th>(D_m) (mm)</th>
<th>(D_{vm}) (mm)</th>
<th>(D_{mm}) (mm)</th>
<th>(N_d) (m(^{-3}))</th>
<th>(W) (g m(^{-3}))</th>
<th>(Z) (dBZ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3550</td>
<td>2.7 %</td>
<td>0.73</td>
<td>0.90</td>
<td>1.27</td>
<td>1623</td>
<td>0.62</td>
<td>81.59</td>
</tr>
<tr>
<td>5</td>
<td>5846</td>
<td>3.6 %</td>
<td>0.98</td>
<td>1.22</td>
<td>1.72</td>
<td>2673</td>
<td>2.52</td>
<td>104.63</td>
</tr>
<tr>
<td>25</td>
<td>9793</td>
<td>4.8 %</td>
<td>1.33</td>
<td>1.64</td>
<td>2.32</td>
<td>4478</td>
<td>10.35</td>
<td>127.79</td>
</tr>
<tr>
<td>50</td>
<td>12303</td>
<td>5.5 %</td>
<td>1.51</td>
<td>1.87</td>
<td>2.64</td>
<td>5626</td>
<td>19.17</td>
<td>137.85</td>
</tr>
<tr>
<td>100</td>
<td>15506</td>
<td>6.4 %</td>
<td>1.72</td>
<td>2.12</td>
<td>2.98</td>
<td>7090</td>
<td>32.26</td>
<td>147.40</td>
</tr>
</tbody>
</table>
Table 4. Rainfall variables of different DSD ($R = 5$ mm h$^{-1}$).

<table>
<thead>
<tr>
<th>Rainfall type</th>
<th>Samples number</th>
<th>Margin probability</th>
<th>$D_m$ (mm)</th>
<th>$D_vn$ (mm)</th>
<th>$D_mm$ (mm)</th>
<th>$N_d$ (m$^{-3}$)</th>
<th>$W$ (g m$^{-3}$)</th>
<th>$Z$ (dBZ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stratiform rain</td>
<td>5846</td>
<td>3.6 %</td>
<td>0.99</td>
<td>1.22</td>
<td>1.72</td>
<td>2673</td>
<td>2.52</td>
<td>104.6</td>
</tr>
<tr>
<td>Mixed stratiform and cumulonimbus rain</td>
<td>3294</td>
<td>4.1 %</td>
<td>1.13</td>
<td>1.49</td>
<td>2.26</td>
<td>1506</td>
<td>2.60</td>
<td>113.44</td>
</tr>
<tr>
<td>Cumulonimbus rain</td>
<td>2748</td>
<td>5.7 %</td>
<td>1.47</td>
<td>2.09</td>
<td>3.40</td>
<td>1257</td>
<td>5.99</td>
<td>133.54</td>
</tr>
</tbody>
</table>
**Table 5.** Relative error of rainfall variables of different DSD ($R = 5\text{ mm h}^{-1}$).

<table>
<thead>
<tr>
<th>Rainfall type</th>
<th>$D_m$ (%)</th>
<th>$D_{vm}$ (%)</th>
<th>$D_{mm}$ (%)</th>
<th>$N_d$ (%)</th>
<th>$W$ (%)</th>
<th>$Z$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stratiform rain</td>
<td>−4.69 %</td>
<td>−2.27 %</td>
<td>−1.03 %</td>
<td>−0.57 %</td>
<td>−3.63 %</td>
<td>−0.64 %</td>
</tr>
<tr>
<td>Mixed stratiform and cumulonimbus rain</td>
<td>−5.53 %</td>
<td>−2.79 %</td>
<td>−1.24 %</td>
<td>2.27 %</td>
<td>−1.99 %</td>
<td>−0.48 %</td>
</tr>
<tr>
<td>Cumulonimbus rain</td>
<td>−7.84 %</td>
<td>−4.25 %</td>
<td>−1.84 %</td>
<td>19.30 %</td>
<td>10.47 %</td>
<td>0.41 %</td>
</tr>
</tbody>
</table>
Table 6. Relative error of stratiform rainfall variables \((R = 10 \text{ mm h}^{-1})\).

<table>
<thead>
<tr>
<th>Disdrometer (sampling size)</th>
<th>Samples number</th>
<th>Margin probability</th>
<th>(D_m) (%)</th>
<th>(D_{vm}) (%)</th>
<th>(D_{mm}) (%)</th>
<th>(N_d) (%)</th>
<th>(W) (%)</th>
<th>(Z) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GBPP-100 (630 mm × 12.6 mm)</td>
<td>11898</td>
<td>8.8 %</td>
<td>−2.79 %</td>
<td>−2.85 %</td>
<td>−2.84 %</td>
<td>%</td>
<td>−8.32 %</td>
<td>−1.49 %</td>
</tr>
<tr>
<td>PARSIVEL (180 mm × 27 mm)</td>
<td>7284</td>
<td>4.3 %</td>
<td>−5.40 %</td>
<td>−2.55 %</td>
<td>−1.01 %</td>
<td>0.14 %</td>
<td>−3.16 %</td>
<td>−0.53 %</td>
</tr>
<tr>
<td>LPM (225 mm × 20 mm)</td>
<td>6744</td>
<td>5.6 %</td>
<td>−1.70 %</td>
<td>−1.66 %</td>
<td>−1.51 %</td>
<td>0 %</td>
<td>−4.89 %</td>
<td>−0.81 %</td>
</tr>
<tr>
<td>OSP (250 mm × 40 mm)</td>
<td>14993</td>
<td>2.8 %</td>
<td>−0.88 %</td>
<td>−0.88 %</td>
<td>−0.83 %</td>
<td>0 %</td>
<td>−2.61 %</td>
<td>−0.44 %</td>
</tr>
<tr>
<td>LAOPS (300 mm × 40 mm)</td>
<td>17991</td>
<td>2.8 %</td>
<td>−0.87 %</td>
<td>−0.87 %</td>
<td>−0.83 %</td>
<td>0 %</td>
<td>−2.57 %</td>
<td>−0.43 %</td>
</tr>
<tr>
<td>2DVD (100 mm × 100 mm)</td>
<td>14993</td>
<td>2.2 %</td>
<td>−0.26 %</td>
<td>−0.32 %</td>
<td>−0.39 %</td>
<td>0 %</td>
<td>−0.95 %</td>
<td>−0.18 %</td>
</tr>
</tbody>
</table>

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Fig. 1. Drop size distribution of different rain ($R = 5 \text{ mm h}^{-1}$).
Fig. 2. Relative errors of stratiform rain variables versus samples number.
Fig. 3. Simulated DSDs and retrieved DSDs of stratiform rainfall of different rainfall intensity.
Fig. 4. Relative error of stratiform rainfall variables versus rainrate.
Fig. 5. Simulated DSDs and retrieved DSDs of rainfall of three types.
Fig. 6. Drop velocity distribution of stratiform rain measured by different instruments ($R = 10 \text{ mm h}^{-1}$).
Effect of sampling variation on error of rainfall variables

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Fig. 7. Drop velocity distribution of different DSDs measured by LAOPS ($R = 10 \text{ mm h}^{-1}$).