Reconstruction of internal gravity wave parameters from radio occultation retrievals of vertical temperature profiles in the Earth atmosphere

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Abstract

The new method for the reconstruction of internal gravity wave (IGW) parameters from a single vertical temperature profile measurement in the Earth atmosphere has been developed. This method does not require any additional information not contained in the profile and may be used for the analysis of profiles measured by various techniques. The criterion for the IGW identification has been formulated and argued. In the case when this criterion is satisfied, then analyzed temperature fluctuations can be considered as wave-induced. The method is based on the analysis of relative amplitude thresholds of the temperature wave field and on the linear IGW saturation theory in which amplitude thresholds are restricted by dynamical (shear) instability processes in the atmosphere. When the amplitude of an internal gravity wave reaches the shear instability limit, energy is assumed to be dissipated in such a way that the amplitude is maintained at the instability limit as the wave propagates upwards. In order to approve the method we have used in situ data of simultaneous balloon high-resolution measurements of the temperature and wind velocity in the Earth stratosphere (France) where a long-period inertia-gravity wave has been detected. Using the temperature data only, we have reconstructed all the measured wave parameters with uncertainties not larger than 30%. An application of the method to the radio occultation data has given the possibility to identify the IGWs in the Earth stratosphere and to determine the magnitudes of key wave parameters such as the intrinsic frequency, amplitudes of vertical and horizontal perturbations of the wind velocity, vertical and horizontal wavelengths, intrinsic vertical and horizontal phase (and group) speeds, kinetic and potential energy, vertical fluxes of the wave energy and horizontal momentum. The obtained results of internal wave studies in the Earth stratosphere deduced from the COSMIC and CHAMP GPS occultation temperature profiles have been presented and discussed.
1 Introduction

Internal gravity waves are small scale waves with the restoring force of buoyancy. They had been examined mostly from a theoretical point of view and treated as “noise” for weather prediction for a long time. However, it was suggested by theoretical works in 1980’s that upward momentum transport by internal gravity waves is a key process to determine the dynamical structure of the middle atmosphere in the height region of 10–100 km such as weak wind layers around the mesopause (Lindzen, 1981; Matsuno, 1982) and in the lower stratosphere (Tanaka and Yamanaka, 1985) in the middle latitude region. With the aid of MST radars and radiosondes with high vertical resolution developed in the same time period, the theoretical prediction was confirmed observationally (Tsuda et al., 1990; Sato, 1994). Then, it is further revealed that the role of IGWs is more important than expected theoretically, namely, in the summer stratosphere (Alexander and Rosenlof, 1996), in the equatorial stratosphere (Sato and Dunkerton, 1997; Dunkerton, 1997) and mesosphere (Mayr et al., 1997). Short and mesoscale IGWs contribute largely to the momentum balance of the stratosphere and dominate in the mesosphere. In addition, the quasi-biennial and semiannual oscillations in equatorial zonal winds are at least partially driven by the convergence of momentum flux (MF) carried by IGWs (Dunkerton, 1997; Garcia and Sassi, 1999; Mayr et al., 1998; Scaife et al., 2000; Giorgetta et al., 2002). However, the IGW parameterization schemes used in global circulation models are still based on important simplifying assumptions (Hines, 1997; Warner and McIntyre, 1999; Medvedev and Klaassen, 2000) and have a number of freely adjustable parameters. Estimates of the MF still rely more on models than on measurements. Therefore an experimental validation of the IGW MF calculated by models is an important issue.

Recently an increasing number of satellite instruments have been appeared for investigation of IGWs. IGW climatologies based on zonal mean values have been published for a Limb Infrared Monitor of the Stratosphere (LIMS) (Fetzer and Gille, 1994). Climatologies were also made for the Microwave Limb Sounder (MLS) on board the
Upper Atmosphere Research Satellite (UARS) (Wu and Waters, 1996a,b; McLandress et al., 2000; Wu, 2001), the meteorological program of the Global Positioning System (GPS/MET) (Tsuda et al., 2000), and the Cryogenic Infrared Spectrometers and Telescopes for the Atmosphere (CRISTA) (Preusse et al., 1999, 2000, 2002). These climatologies are all based on gravity wave temperature variances and, in most cases, give estimates of either horizontal or vertical wavelengths. However, none of these analyses have provided simultaneous estimates of both horizontal and vertical wavelength, so they cannot be used to estimate intrinsic frequency and MF. The direct deduction of momentum flux requires the simultaneous measurement of horizontal and vertical wind. Only a very few of measurement techniques are capable of this. Using the dispersion and polarization relations of IGWs, Ern et al. (2004) have shown that the momentum flux can be estimated from temperature amplitudes if the horizontal and vertical wavelengths of the waves are known.

There is one general difficulty inherent to all measurements of IGWs. Measured is a state variable of the atmosphere, as for instance wind or temperature, and one has to determine from this measurement which part of the “signal” is attributed to IGWs. The general approach is to isolate smaller scale variations from a slowly varying background and to declare that these perturbations are the signal imprinted by IGWs. If the measurements provide time-series at a given location, as for instance lidar or radar measurements do, separation can be made due to frequency analyses. Any scale separation approach has of course two problems. First, not all small scale variations in the atmosphere are IGWs and, second, not all IGWs are of small scales. Observed variations can alternatively be connected with internal gravity waves, turbulence or stable layers. For the correct interpretation of scientific results it is necessary to have the IGW identification criterion, and in the case when this criterion is satisfied, then analyzed variations can be considered as wave-induced.

Atmospheric observation by satellites with GPS occultation method is powerful to examine a gravity wave distribution around the planet with almost uniform quality. Analysis of the temperature variations found from radio occultation (RO) experiments allows
to measure the statistical characteristics of internal waves in the atmosphere (Steiner and Kirchengast, 2000; Tsuda et al., 2000; Tsuda and Hocke, 2002). The advantages of RO observations are that this technique can furnish atmospheric profiles with global spatial coverage and high vertical resolution, and under all weather conditions (Liou et al., 2003, 2004). But before now it was supposed that because in radio occultation measurements the observed quantities are only temperature and atmospheric density, it is impossible to estimate IGW parameters such as intrinsic frequency and phase velocities that are necessary to quantify the gravity wave effects. In this connection we have developed the new method for the IGW identification and reconstruction of internal gravity wave parameters from the single vertical temperature or density profile measurement in a planetary atmosphere (Gubenko et al., 2008, 2011).

The aim of this work is the:

1. formulation and argumentation of the identification criterion for an internal gravity wave which saturates via shear instability;

2. development and improvement of the method for determination the intrinsic frequency and other wave parameters from a single vertical temperature profile;

3. verification of the developed technique on the base of simultaneous in situ balloon measurements of the temperature and wind velocity in the Earth’s stratosphere where the saturated internal wave has been detected;

4. examples of the practical application of the proposed technique on radio occultation retrievals of temperature profiles in the Earth’s atmosphere.

2 Theoretical relationships

Assuming an adiabatic inviscid process and small scale motions, the perturbation equations for atmospheric motions can be derived from the fundamental equations for the atmospheric dynamics such as the equation of motion, first law of thermodynamics,
continuity equation and equation of state for an ideal gas (Fritts and Alexander, 2003). If an internal gravity wave is a plane wave, we can derive the dispersion relation of a gravity wave when $f^2 < \omega^2 < N^2$ and $m^2 \gg 1/(4H^2)$, as follows:

$$\frac{\omega^2}{k^2 + l^2} = \frac{N^2}{m^2} \frac{1 - \omega^2/N^2}{1 - f^2/\omega^2},$$  \hspace{1cm} (1)$$

where $H \approx 7$ km is the scale height in the middle atmosphere, $N$ is the Brunt-Vaisala, $f$ the inertial, and $\omega$ the intrinsic frequency (frequency observed in the frame moving with the background flow) of the wave. The inertial frequency (Coriolis parameter) $f$ is defined as $f = 2\Omega \cdot \sin \phi$, where $\Omega = 7.292 \times 10^{-5}$ rad s$^{-1}$ is the Earth rotation rate and $\phi$ is latitude. The intrinsic frequency can be written as $\omega = (k^2 + l^2)^{1/2}|c - \bar{u}|$, where $c$ is the ground-based horizontal phase velocity relative to observer and $\bar{u}$ is the mean background horizontal wind component in the direction of wave propagation. The parameters $k$, $l$ and $m$ represent the horizontal and vertical wave numbers. We can make $l = 0$ by taking the x-axis oriented along the direction of propagation of the wave.

The polarization relations for gravity waves can be derived from the basic equations as follows:

$$v' = -i \cdot \frac{f}{\omega} \cdot u',$$  \hspace{1cm} (2)$$

$$w' = -\frac{k}{m} \cdot u',$$  \hspace{1cm} (3)$$

where $u'$ and $v'$ are the complex perturbations for the parallel and perpendicular components of wave-induced horizontal wind relative to the wave propagation direction, $w'$ is the complex perturbation for the wave-induced vertical wind. The prime (') and the overbar denote the perturbation and mean quantities, respectively. There are simple relationships between the complex and real physical perturbations. For example, the real perturbation correspondent to $u'$ is $\text{Re} \ u'$, and the real perturbation amplitude $|u'|$ is $[(\text{Re} \ u')^2 + (\text{Im} \ u')^2]^{1/2}$. Equation (2) shows that the phase difference between $u'$ and
\( \nu' \) is 90°, and the ratio of amplitudes \( \left| \nu' \right| / \left| u' \right| \) is proportional to \( f/\omega \), indicating that the tip of the horizontal wind vector of a gravity wave draws an elliptical locus, and that the gravity wave is a transverse wave.

For an atmosphere in hydrostatic equilibrium where the fractional pressure fluctuations can be neglected relative to those in temperature and density, the amplitudes of normalized density and temperature perturbations can be computed from the ideal gas law and are approximately equal for a wave (Holton, 1992; Gardner et al., 1998). For medium-frequency or low-frequency waves \( (f^2 < \omega^2 \ll N^2) \), the following approximate polarization relations are simultaneously valid in this case (Eckermann et al., 1995; Gavrilov, 1997):

\[
|u'| \approx \frac{g}{N} \cdot \left| \frac{\rho'}{\bar{\rho}} \right| \cdot \left( 1 - \frac{f^2}{\omega^2} \right)^{-1/2}, \tag{4}
\]

\[
|u'| \approx \frac{g}{N} \cdot \left| \frac{T'}{\bar{T}} \right| \cdot \left( 1 - \frac{f^2}{\omega^2} \right)^{-1/2}, \tag{5}
\]

where \( g \) is the acceleration due to gravity, \( \left| \frac{\rho'}{\bar{\rho}} \right| \) and \( \left| \frac{T'}{\bar{T}} \right| = \left| \frac{T'}{\bar{T}} \right| \) are the amplitudes of normalized density and temperature perturbations, respectively, and \( \left| u' \right| \) is the amplitude of velocity perturbation in the direction of horizontal wave vector component.

A more detailed discussion of the restrictions connected with Eqs. (4) and (5) can be found in the work of Pavelyev et al. (2004). Observations of wind and temperature fluctuations in the middle atmosphere, attributed to internal gravity waves, have shown the amplitudes to increase with altitude, but more slowly than implied by the decrease in density in the absence of dissipation (Fritts, 1984). Linear saturation theory was first used to address gravity wave instability and turbulence production, and the assumption implicit in this theory is that any wave amplitude in excess of a threshold value will lead to instability and the production of turbulence that acts to prevent further growth of the wave amplitude.
The relative amplitude threshold $a$ (scaled by the intrinsic phase speed, see below) is defined as the wave amplitude required for instability. It is assumed that the nominal threshold amplitudes for dynamical and convective instability are those which yield $Ri < 1/4$ and $Ri < 0$, respectively, with the local Richardson number ($Ri$) defined to be (Fritts and Rastogi, 1985):

$$Ri = \frac{N^2}{(Re u'_z)^2 + (Re v'_z)^2}, \quad (6)$$

where $Re u'_z$ and $Re v'_z$ are the local wind shears along a vertical axis $z$. For most intrinsic frequencies the two threshold amplitudes are nearly identical and equal to unity, departing significantly only for $\omega \sim f$. It was suggested by Fritts (1984) and shown by Dunkerton (1984) and Fritts and Rastogi (1985) that wave motions can be dynamically unstable (as measured along a vertical axis) at substantially smaller wave amplitudes for intrinsic frequencies near the inertial frequency due to the transverse shear in the velocity field of such motions. In the case when $\omega^2 \ll N^2$, the theoretical amplitude threshold $a$ for shear instability assuming a minimum Richardson number of 1/4 is given by (Fritts and Rastogi, 1985; Fritts, 1989):

$$a = \frac{|u'|}{|c - \bar{u}|} = \frac{2 \cdot \left(1 - f^2/\omega^2\right)^{1/2}}{1 + \left(1 - f^2/\omega^2\right)^{1/2}}. \quad (7)$$

As a function of $f/\omega$, this threshold $a$ falls from unity to zero when the ratio $f/\omega$ changes from zero ($\omega^2 \gg f^2$) to unity ($\omega \sim f$). Thus when $f^2 < \omega^2 \ll N^2$, the following condition for $a$ is valid:

$$1 > a > 0. \quad (8)$$
3 Identification of IGWs and determination of wave parameters

In order to identify the observed wavelike fluctuations as wave-induced we, initially, determine the experimental value of relative amplitude threshold $a_e$. If the $a_e$ value satisfies the condition (Eq. 8), then we can assume that observed fluctuations are due to a wave. In this case, the Eq. (7) gives the possibility to determine the $f/\omega$ ratio.

Note that the criterion (Eq. 8) for the identification of IGWs from observations is necessary, but not sufficient, because the observed perturbations may result from processes other than gravity waves. For example, these may be due to thin stable layers or random measurement errors. However, inspection of the raw data shows that stable layers produce, as a rule, large impulsive bursts with small enough vertical scales. The role of measurement noise for gravity wave characteristics obtained from radio occultation temperature retrievals of GPS was discussed by Marquardt and Healy (2005). They showed that for realistic noise levels, temperature fluctuations can be safely interpreted as originating from small scale atmospheric waves in the altitude region below 30 km and for wavelengths above 2 km. For noise levels typical for GPS/MET, standard deviations are below or at most in the order of 1 K below 30 km altitude, and the contribution of noise to the power spectral density is by two orders of magnitudes smaller for vertical wavelengths above 2 km (Marquardt and Healy, 2005). This consideration leads us to the conclusion that observed temperature fluctuations in the low stratosphere may generally be related to gravity waves when the $a_e$ value of relative amplitude threshold obeys the relation:

$$1 > a = a_e > 0.$$  \hspace{1cm} (9)

For the above mentioned reasons, the criterion (Eq. 9) for the IGW identification may be considered not only necessary, but sufficient at least below 30 km altitude where the noise appears unable to account for observed fluctuations. In the opposite case when $a_e > 1$, the observed fluctuations can also be connected with a thin stable layer or turbulence. The main idea of the experimental determination of the relative amplitude...
threshold $a_e$ is as follows: by using the definition of intrinsic horizontal phase speed $|c - \bar{u}|$ and Eq. (1), in the case when $\omega^2 \ll N^2$, it is easy to obtain the relation:

$$|c - \bar{u}| = \frac{\omega}{|k_h|} = \frac{N}{|m|} \cdot \left(1 - f^2/\omega^2\right)^{-1/2} = N \frac{\lambda_z}{2\pi} \cdot \left(1 - f^2/\omega^2\right)^{-1/2},$$

(10)

where $k_h = (k^2 + l^2)^{1/2}$ is the modulus of horizontal wave number, $\lambda_z = 2\pi/|m|$ is the vertical wavelength. Substituting the expression for $|u'|$ from Eqs. (4) or (5) into Eq. (7), and taking into account Eqs. (9) and (10), one can find:

$$a_e = \frac{g|m|}{N^2} \cdot \frac{\rho'}{\overline{\rho}} = \frac{2\pi \cdot g}{\lambda_z N^2} \cdot \frac{|\rho'|}{\overline{\rho}} = \frac{2 \left(1 - f^2/\omega^2\right)^{1/2}}{1 + \left(1 - f^2/\omega^2\right)^{1/2}},$$

(11)

$$a_e = \frac{g|m|}{N^2} \cdot \frac{T'}{\overline{T}} = \frac{2\pi \cdot g}{\lambda_z N^2} \cdot \frac{|T'|}{\overline{T}} = \frac{2 \left(1 - f^2/\omega^2\right)^{1/2}}{1 + \left(1 - f^2/\omega^2\right)^{1/2}}.$$

(12)

Because all parameters included in the left part of Eqs. (11) and (12) can be restored from RO measurements of density or temperature, so it enables to determine experimentally the relative amplitude threshold $a_e$.

When the observed wavelike variations $\rho'/\overline{\rho}$ or $T'/\overline{T}$ are positively identified and the experimental value $a_e$ can be found, the Eq. (11) or Eq. (12) allows to determine the intrinsic frequency of the internal gravity wave:

$$\omega = \frac{f}{2} \cdot \frac{2 - a_e}{(1 - a_e)^{1/2}},$$

(13)

In addition, this simple relation holds:

$$\left(1 - f^2/\omega^2\right)^{1/2} = \frac{a_e}{2 - a_e}.$$

(14)
On substituting Eq. (14) into Eq. (10), for intrinsic horizontal phase speed \( |C_{ph}^{in}| = |c - \bar{u}| \) we can obtain:

\[
|C_{ph}^{in}| = |c - \bar{u}| = \frac{N}{|m|} \cdot \frac{2 - a_e}{a_e} = \frac{\lambda_z N}{2 \pi} \cdot \frac{2 - a_e}{a_e}.
\] (15)

According to the definition of intrinsic vertical phase speed \( |C_{pz}^{in}| \), and using the Eq. (13), we have:

\[
|C_{pz}^{in}| = \frac{\omega}{|m|} = \frac{f}{2 |m|} \cdot \frac{2 - a_e}{(1 - a_e)^{1/2}} = \frac{\lambda_z f}{4 \pi} \cdot \frac{2 - a_e}{(1 - a_e)^{1/2}}.
\] (16)

Taking into account the Eqs. (7), (9) and (15), the amplitude of velocity perturbations \( |u'| \) may be found:

\[
|u'| = a_e |c - \bar{u}| = \frac{N}{|m|} \cdot (2 - a_e) = \frac{\lambda_z N}{2 \pi} \cdot (2 - a_e).
\] (17)

One can find the other perturbation amplitudes, \( |v'| \) and \( |w'| \), by using the Eqs. (2), (3), (10), (13), (15) and (17):

\[
|v'| = \frac{f}{\omega} \cdot |u'| = 2 \frac{N}{|m|} \cdot (1 - a_e)^{1/2} = \frac{\lambda_z N}{\pi} \cdot (1 - a_e)^{1/2},
\] (18)

\[
|w'| = \frac{|k_h|}{|m|} \cdot |u'| = \frac{f \cdot a_e}{2 |m|} \cdot \frac{2 - a_e}{(1 - a_e)^{1/2}} = \frac{\lambda_z f \cdot a_e}{4 \pi} \cdot \frac{2 - a_e}{(1 - a_e)^{1/2}}.
\] (19)

The horizontal wave number \( |k_h| \) and the horizontal wavelength \( \lambda_h \) can be found from Eqs. (13) and (15):

\[
|k_h| = \frac{\omega}{|c - \bar{u}|} = \frac{|m|}{2} \cdot \frac{f \cdot a_e}{N (1 - a_e)^{1/2}} = \frac{\pi \cdot f}{\lambda_z N} \cdot \frac{a_e}{(1 - a_e)^{1/2}}.
\] (20)
\[ \lambda_h = \frac{2 \pi}{|k_h|} = 4 \pi \frac{N}{f} \cdot \frac{(1 - a_e)^{1/2}}{a_e \cdot |m|} = 2 \frac{\lambda_z N}{f} \cdot \frac{(1 - a_e)^{1/2}}{a_e} . \]  

The relative amplitude threshold and other wave parameters can be experimentally determined by this method when the altitude profile of the Brunt-Vaisala frequency is known. We computed the Brunt-Vaisala frequency, \( N \), by evaluating (Lindzen, 1981):

\[ N^2 = \frac{g}{T} \cdot \left( \frac{\partial T}{\partial z} + \frac{g}{c_p} \right) , \]  

where \( g/c_p = 9.8 \times 10^{-3} \text{ K m}^{-1} \) is the adiabatic temperature lapse rate, \( c_p \) is the specific heat capacity at constant pressure. Let us consider an atmosphere with the small wave-induced harmonic perturbations \( T'(z) \) of the constant basic state temperature field \( T_b(z) = \bar{T}(z) \). If \( |T'| \) is the wave amplitude (\( |T'| \ll T_b \)) and \( m \) is the vertical wave number, then the atmospheric temperature \( T(z) \) is given by:

\[ T(z) = T_b(z) + |T'| \cdot \cos \left[ m (z - z_0) + \psi \right] , \]

where \( \psi \) is the wave phase for the reference level \( z_0 \). By substituting \( T(z) \) from Eq. (23) into Eq. (22) and taking into account that \( T(z) \approx T_b(z) \), we have:

\[ N^2(z) = \frac{g}{T_b(z)} \cdot \left( \frac{\partial T_b(z)}{\partial z} + \frac{g}{c_p} \right) + \frac{g m |T'|}{T_b(z)} \cdot \cos \left[ m (z - z_0) + \psi + \pi/2 \right] . \]

The first term in Eq. (24) is the simple background estimate of \( N_b^2 = N_{\text{med}}^2 \) and it may be used for substitution into Eq. (12) in order to determine \( a_e \). It is clear from Eq. (24) that the vertical profile \( N_b^2(z) \) can be calculated by application of Eq. (22) to the background profile \( T_b(z) \). The second term in Eq. (24) describes absolute fluctuations of \( N^2 \) associated with the temperature perturbations. It is easy to find from Eq. (24) that the amplitude \( A_{N^2}^\text{rel} \) of relative fluctuations of the Brunt-Vaisala frequency squared \((N^2 - N_b^2)/N_b^2\) is given by:
where $N^2_{\text{min}} = \min N^2(z)$. Thus, the magnitude of relative amplitude threshold $a_e$ may be expressed in terms of the temperature or $N^2$ and these two estimates of the wave amplitude $a_e$ must be in the mutual agreement. Note that the local shear instability and its associated turbulence in the Earth's stratosphere grow in the vertical shear of the transverse velocity component which maximizes at the local minimum of Brunt-Vaisala frequency (Fritts and Rastogi, 1985; Fritts, 1989; Dunkerton, 1989).

4 Reconstruction of the wave energy and momentum fluxes, and verification of the developed technique

With the wave parameters which determinations were considered in Sect. 3 it is possible to reconstruct such important characteristics for IGW activity as the kinetic, potential and total wave energy, and also wave fluxes of the energy and horizontal momentum per mass unit. It is known that the total wave energy $E$ consists of two parts: kinetic, $E_k$, and potential, $E_p$, which are given by the following expressions (Gill, 1982):

$$E = E_k + E_p = \frac{1}{2} |w'|^2 \left( 1 + \tan^2 \phi' \right) = \frac{1}{2} \left( |w'|^2 + |u'|^2 \right),$$ (26)

$$E_k = \frac{1}{2} \left[ u'^2 + v'^2 + w'^2 \right] = \frac{1}{4} \left[ |u'|^2 + |v'|^2 + |w'|^2 \right] = E \cdot \frac{1+(f/\omega)^2 \sin^2 \phi'}{2},$$ (27)

$$E_p = \frac{1}{2} \frac{g^2}{N^2} \left( \frac{T'}{T} \right)^2 = \frac{1}{4} \frac{g^2}{N^2} \left| \frac{T'}{T} \right|^2 = E \cdot \frac{1-(f/\omega)^2 \sin^2 \phi'}{2}. \quad (28)$$
Here and in the following, for simplicity, the tokens Re before primed quantities are omitted, and primed ones mean real perturbations to the background state; the overbar denotes a spatial average over a vertical wavelength $\lambda_z$; an angle $\phi'$ is that between the wave vector direction and horizontal plane, and the following simple relation for $\phi'$ holds:

$$\tan \phi' = \frac{|m|}{|k_h|} = \frac{\lambda_h}{\lambda_z}. \quad (29)$$

For IGWs with $\omega^2 \ll N^2$, the Eqs. (26)–(28) can be easily obtained and verified by using the dispersion and polarization Eqs. (1)–(3), (5), the Eq. (29) and taking into account the fact that the average of some harmonic perturbation squared over a vertical wavelength is equal to the corresponding amplitude squared divided by 2 (for example, $u'^2 = |u'|^2 / 2$). The ratio $p$ of the kinetic to potential energy may be found from Eqs. (27) and (28) with the aid of the dispersion Eq. (1):

$$p = \frac{E_k}{E_p} = \frac{\omega^2 + f^2 \sin^2 \phi'}{\omega^2 - f^2 \sin^2 \phi'} = 1 + 2 \frac{f^2}{N^2} \tan^2 \phi'. \quad (30)$$

The vertical, $|F_z|$, and horizontal, $|F_h|$, fluxes of wave energy per mass unit may be written as (Gill, 1982):

$$|F_z| = |C_{gz}^{in}| \cdot E, \quad (31)$$

$$|F_h| = |C_{gh}^{in}| \cdot E, \quad (32)$$

where $|C_{gz}^{in}|$ and $|C_{gh}^{in}|$ are the modules of the intrinsic vertical and horizontal components of the group speed, correspondingly. By using the definitions of $|C_{gz}^{in}|$ and...
\[ |C_{gh}^{in}|, \text{ and the dispersion Eq. (1) with } \omega^2 \ll N^2, \text{ one can find:} \]

\[ |C_{gh}^{in}| = \left| \frac{\partial \omega}{\partial k_h} \right| = \left| C_{ph}^{in} \right| \left( 1 - \frac{f^2}{\omega^2} \right) = \frac{|k_h|N}{m} \cdot \sqrt{1 - \frac{f^2}{\omega^2}}. \quad (33) \]

\[ |C_{gh}^{in}| = \left| \frac{\partial \omega}{\partial k_h} \right| = \left| C_{ph}^{in} \right| \left( 1 - \frac{f^2}{\omega^2} \right) = \frac{N}{|m|} \cdot \sqrt{1 - \frac{f^2}{\omega^2}}. \quad (34) \]

Taking into account the definition of the total vertical flux of horizontal momentum, \( |F_{ph}| \), due to IGW and the polarization Eqs. (2), (3), the following expression for \( |F_{ph}| \) can be found:

\[ |F_{ph}| = \left| u' \cdot w' \right| = \frac{|u'| \cdot |w'|}{2} = \frac{|k_h|}{m} \cdot \frac{|u'|^2}{2}. \quad (35) \]

For the experimental verification of the analysis technique proposed, we used the results of the simultaneous temperature and wind velocity measurements (France) obtained in a high-resolution balloon experiment (Cot and Barat, 1986), where a nearly monochromatic and long-period wave propagating upward in the stratosphere was identified. From all the data, Cot and Barat (1986) have estimated the mean characteristics of the wave, such as the vertical and horizontal wavelengths, ratio \( f/\omega \), vertical and horizontal wave-induced velocity amplitudes, intrinsic vertical and horizontal phase speeds, temperature perturbation amplitude. By using the temperature data only, we reconstructed the ratio \( f/\omega \) and other wave parameters. In Table 1 we give observed wave parameters (Cot and Barat, 1986), our reconstruction results, and relative deviations of observed and reconstructed wave parameters. An comparison of the observed and reconstructed wave parameters shows a good agreement between them. It is seen from the values given in Table 1 that the relative deviations of the reconstructed parameters are not larger than 31%. The best agreement is found for such parameters as
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5 Experimental derivation of IGW parameters from RO temperature data

In order to demonstrate examples of practical application of the proposed technique, we used validated temperature data obtained from CHAMP and COSMIC GPS radio occultation where wavelike periodic structures in the lower stratosphere were visible. The altitude sampling of the original four profiles is unequal, and for providing equidistant altitude steps required for the analysis, the data were smoothed and interpolated to a regular 100-m grid. The altitude intervals \( L \) of wave observations were restricted by stratospheric levels from 15 to 36 km where the RO temperature data have the high vertical resolution \( \delta h \) and accuracy \( \delta T \). It is known that below 30–35 km within the upper troposphere and lower stratosphere, the influence of the ionosphere error is negligible and the temperature error is less than 1 K and vertical resolution is better than 1 km (Hocke, 1997; Steiner and Kirchengast, 2000). In the case of measurement temperature errors, we have, instead of the IGW criterion (Eq. 9), some additional limitations for the experimental magnitude \( a_e \), namely (Gubenko et al., 2008, 2011):

\[
1 > (1 + X) a_e > a_e > (1 - X) a_e > 0, \tag{36}
\]

where \( X = |\delta a_e/a_e| \) is the relative uncertainty in the amplitude threshold \( a_e \) and \( \delta a_e \) is the standard deviation. The expression (Eq. 36) states that not only \( a_e \), but also \( (a_e - \delta a_e) \) and \( (a_e + \delta a_e) \) satisfy the IGW criterion (Eq. 9). This implies that we have reliable estimates of wave parameters because the relative uncertainties and standard deviations of reconstructed IGW parameters are limited in this case. The detailed discussion of errors for the estimated wave parameters and analytical expressions for
their determination from the real measurement uncertainties $\delta T$ and $\delta h$ can be found in the work of Gubenko et al. (2008).

Let us consider the CHAMP GPS radio occultation retrievals of temperature profiles in the Earth’s stratosphere. Figure 1 shows the example of IGW observations in the middle latitudes at altitudes from 15.5 to 24 km. The RO measurements were made on 13 January 2002 in the atmospheric region with the latitude 37.09° N and longitude 102.96° E. In successive panels of Fig. 1, from left to right, are plotted the altitude profiles of temperature, normalized temperature variation, and Brunt-Vaisala frequency squared $N^2$. The solid line (left panel) is the raw temperature data, $T$, and the dotted line is the background temperature profile, $T_b$, determined by a least-squares cubic polynomial curve fit within the altitude interval $L = 8.5$ km of wave observations. Based on these raw and background profiles, we computed the corresponding original, $N^2$, (solid line) and mean, $N_b^2$, (dotted line) profiles of the Brunt-Vaisala frequency squared (right panel) with the aid of the Eq. (22). We used a low-pass digital filter with a cutoff at 2.0 km to calculate the smoothed profile (dotted line, middle panel) of normalized temperature variations from raw data (solid line, middle panel). In order to compute the normalized temperature variation profile ($T'/T_b$), the standard expression $T'/T_b = (T - T_b)/T_b$ was used. The altitude profile $T'(z)/T_b(z)$ (middle panel) was used for evaluation of the normalized temperature perturbation amplitude $|\hat{T}| = |T'|/T_b$ and vertical wavelength $\lambda_z$. It was found that the value of the normalized amplitude is equal to $10^{-2}$ and the vertical wavelength is equal to 3.2 km. For the calculation of the experimental magnitude of the relative amplitude threshold $a_e$ it is necessary to ensure a good estimate of the Brunt-Vaisala frequency squared. Figure 1 shows plots of $N^2$ and $N_b^2$ profiles (right panel) for the sounded region and confirms that the $N_b^2$ value is weakly sensitive to actual thermal structure and well represented by a constant for the investigated altitude range. For the determination of the experimental value $a_e$, we thus took reasonable constant value $N_b^2 = 4.35 \times 10^{-4}$ rad$^2$ s$^{-2}$. The local minimum of the Brunt-Vaisala frequency squared $N_{\text{min}}^2 = \min N^2 \approx 2 \times 10^{-4}$ rad$^2$ s$^{-2}$ is shown in
Fig. 1 by arrow. Substituting these estimates of $N^2_b$, $\lambda_z$ and $|T'| / T_b$ into Eq. (12) it was found that $a_e = 0.44$. In order to find uncertainties for estimated IGW parameters, we adopted that the original temperature data were obtained with the accuracy $\delta T = 0.5 \text{ K}$ and vertical resolution $\delta h = 1200 \text{ m}$. Following Gubenko et al. (2008) we found that the relative uncertainty of $a_e$ is equal to $X = 0.32$, and value $a_e = 0.44$ satisfies to the modified IGW criterion (Eq. 36). Thus, the strong wavelike oscillations of $T$ and $N^2$ are probably caused by an internal gravity wave (see IGW parameters and their uncertainties in Table 2).

Figure 2 demonstrates the example of IGW observations in the high latitudes at altitudes from 23.0 to 27.5 km. The CHAMP GPS RO measurements were conducted on 13 January 2002 in the atmospheric region with the latitude 67.18° N and longitude 169.42° E. Figure 3 shows the example of IGW observations in the high latitudes at altitudes from 19.9 to 26.2 km. The CHAMP GPS RO measurements were conducted on 17 January 2002 in the atmospheric region with the latitude 68.44° N and longitude 14.18° W. Figure 4 demonstrates the example of IGW observations in the middle latitudes at altitudes from 28.3 to 36.0 km. These COSMIC GPS RO measurements were made on 12 June 2009 in the atmospheric region with the latitude 54.92° N and longitude 166.85° E. Descriptions in Figs. 2–4 are the same as in Fig. 1.

From the temperature data, we compute the wave parameters for the four sounded regions. In Table 2 we have summarized all the wave parameters and their uncertainties if these are less than 100%. Thus we have shown that the wave characteristics can be obtained from a single temperature profile in the case of positive IGW identification.
6 Conclusions

1. The new method for the determination of IGW parameters from a single vertical temperature or density profile measurement in a planetary atmosphere has been developed. This method does not require any additional information not contained in the profile and may be used for the analysis of profiles measured by various techniques.

2. The criterion for the IGW identification has been formulated and argued. In the case when this criterion is satisfied then analyzed temperature or density fluctuations can be considered as wave-induced.

3. The experimental verification of the analysis technique proposed was made by using the results of simultaneous temperature and wind velocity in situ measurements obtained in a high-resolution balloon experiment (Cot and Barat, 1986), where a nearly monochromatic and long-period wave propagating upward in the stratosphere was identified. Using the temperature data only, we reconstructed all the measured wave parameters with uncertainties not larger than 30%.

4. The suggested method is most effective in the case of low intrinsic IGW frequencies when the experimentally determined amplitude threshold $a_e$ appreciably differs from unity.

5. The analysis of radio occultation data on the basis of proposed method gave the possibility to identify the IGWs in the Earth’s atmosphere and to determine the magnitudes of key wave parameters such as the intrinsic frequency, amplitudes of vertical and horizontal perturbations of the wind velocity, vertical and horizontal wavelengths, intrinsic vertical and horizontal phase (and group) speeds, kinetic and potential energy, vertical fluxes of the wave energy and horizontal momentum.

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References


Table 1. Wave parameters observed by Cot and Barat (1986) and those reconstructed from temperature data. Relative deviations between observed and reconstructed wave parameters are indicated.

<table>
<thead>
<tr>
<th>Wave parameters observed by Cot and Barat (1986)</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<tbody>
<tr>
<td>1.0 1.0 231.5 0.02 – 0.80 1.25 5.2 3.3 260 0.020 0.013</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic parameters for reconstruction</td>
<td>Parameters reconstructed from temperature data</td>
<td>1.0 1.0 231.5 0.02 0.67 0.86 1.16 6.3 4.2 341 0.018 0.012</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative deviations (%) of observed and reconstructed wave parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>– 7.5 7.2 21.2 27.3 31.2 10.0 7.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</table>
Table 2. Wave parameters found from CHAMP & COSMIC GPS radio occultation retrievals of temperature profiles in four regions of the Earth’s atmosphere. The coordinates of sounded regions, time and altitude intervals of observations are indicated. Uncertainties of determined parameters are also shown if these <100%.

<table>
<thead>
<tr>
<th>Satellite</th>
<th>IGW</th>
<th>parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CHAMP</td>
<td>06:03 UT</td>
<td>15:37 UT</td>
<td>15:27 UT</td>
</tr>
<tr>
<td>COSMIC</td>
<td>37.09° N</td>
<td>67.18° N</td>
<td>68.44° N</td>
</tr>
<tr>
<td></td>
<td>102.96° E</td>
<td>169.42° E</td>
<td>14.18° W</td>
</tr>
<tr>
<td></td>
<td>[15.5:24.0] km</td>
<td>[23.0:27.5] km</td>
<td>[19.9:26.2] km</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Satellite</th>
<th>CHAMP</th>
<th>CHAMP</th>
<th>CHAMP</th>
<th>COSMIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$, km</td>
<td>8.5</td>
<td>4.5</td>
<td>6.3</td>
<td>7.7</td>
<td></td>
</tr>
<tr>
<td>$\lambda_z$, km</td>
<td>3.2 ± 0.7</td>
<td>1.7 ± 0.7</td>
<td>2.1 ± 0.7</td>
<td>3.4 ± 0.8</td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>T</td>
<td>\times 10^{-3}$, rel. un.</td>
<td>10.0 ± 1.4</td>
<td>4.9 ± 1.4</td>
<td>7.8 ± 1.4</td>
</tr>
<tr>
<td>$N_b$, $10^{-4}$ rad$^2$ s$^{-2}$</td>
<td>4.35 ± 0.64</td>
<td>3.75 ± 0.80</td>
<td>4.50 ± 0.96</td>
<td>5.00 ± 0.55</td>
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<tr>
<td>$a_e$, rel. un.</td>
<td>0.44 ± 0.14</td>
<td>0.48 ± 0.27</td>
<td>0.50 ± 0.22</td>
<td>0.45 ± 0.13</td>
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<tr>
<td>$f/\omega$, rel. un.</td>
<td>0.96 ± 0.03</td>
<td>0.95 ± 0.08</td>
<td>0.94 ± 0.07</td>
<td>0.96 ± 0.03</td>
<td></td>
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<tr>
<td>$\omega$, $10^{-4}$ rad$^{-1}$ s$^{-1}$</td>
<td>0.92 ± 0.03</td>
<td>1.42 ± 0.12</td>
<td>1.44 ± 0.11</td>
<td>1.25 ± 0.04</td>
<td></td>
</tr>
<tr>
<td>$T^n$, hours</td>
<td>19.0 ± 0.7</td>
<td>12.3 ± 1.0</td>
<td>12.1 ± 0.9</td>
<td>14.0 ± 0.5</td>
<td></td>
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<tr>
<td>$\tan \phi'$, rel. un.</td>
<td>800 ± 436</td>
<td>434</td>
<td>437 ± 353</td>
<td>611 ± 319</td>
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<tr>
<td>$90^\circ - \phi'$, deg</td>
<td>0.07 ± 0.04</td>
<td>0.13</td>
<td>0.13 ± 0.11</td>
<td>0.09 ± 0.05</td>
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<tr>
<td>$</td>
<td>C_{phh}</td>
<td>$, m s$^{-1}$</td>
<td>37.4 ± 17.3</td>
<td>16.4 ± 14.2</td>
<td>21.2 ± 14.3</td>
</tr>
<tr>
<td>$</td>
<td>u</td>
<td>$, m s$^{-1}$</td>
<td>16.5 ± 4.2</td>
<td>7.9 ± 3.8</td>
<td>10.7 ± 4.0</td>
</tr>
<tr>
<td>$</td>
<td>v</td>
<td>$, m s$^{-1}$</td>
<td>15.9 ± 4.3</td>
<td>7.5 ± 3.9</td>
<td>10.1 ± 4.1</td>
</tr>
<tr>
<td>$</td>
<td>C_{php}</td>
<td>$, $10^{-3}$ m s$^{-1}$</td>
<td>46.7 ± 10.9</td>
<td>37.8 ± 16.8</td>
<td>48.6 ± 16.4</td>
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<td>$</td>
<td>w</td>
<td>$, $10^{-3}$ m s$^{-1}$</td>
<td>20.7 ± 8.6</td>
<td>18.1 ± 14.0</td>
<td>24.5 ± 14.8</td>
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<tr>
<td>$\lambda_h$, km</td>
<td>2560 ± 1260</td>
<td>727 ± 680</td>
<td>927 ± 684</td>
<td>2080 ± 970</td>
<td></td>
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<tr>
<td>$</td>
<td>C_{ghh}</td>
<td>$, m s$^{-1}$</td>
<td>3.0 ± 1.4</td>
<td>1.6 ± 1.4</td>
<td>2.4 ± 1.6</td>
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<tr>
<td>$</td>
<td>C_{gp}</td>
<td>$, $10^{-3}$ m s$^{-1}$</td>
<td>3.8 ± 2.7</td>
<td>3.7</td>
<td>5.5</td>
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<tr>
<td>$E_p$, m$^2$ s$^{-2}$</td>
<td>5.5 ± 1.8</td>
<td>1.5 ± 0.9</td>
<td>3.3 ± 1.4</td>
<td>7.5 ± 1.9</td>
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</tr>
<tr>
<td>$E$, m$^2$ s$^{-2}$</td>
<td>137 ± 70</td>
<td>30.8 ± 29.8</td>
<td>57.3 ± 43.0</td>
<td>175 ± 89</td>
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<td>$p = E_k/E_p$, rel. un.</td>
<td>23.7 ± 15.0</td>
<td>19.2</td>
<td>16.6 ± 15.2</td>
<td>22.3 ± 13.2</td>
<td></td>
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<tr>
<td>$</td>
<td>F_{phh}</td>
<td>$, m$^3$ s$^{-3}$</td>
<td>413 ± 286</td>
<td>50</td>
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<td>$</td>
<td>F_{php}</td>
<td>$, m$^3$ s$^{-3}$</td>
<td>0.52 ± 0.46</td>
<td>0.12</td>
<td>0.32</td>
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<tr>
<td>$</td>
<td>F_{ph}</td>
<td>$, m$^3$ s$^{-2}$</td>
<td>0.17 ± 0.08</td>
<td>0.07 ± 0.06</td>
<td>0.13 ± 0.09</td>
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</table>
Fig. 1. Example of IGW observations in the middle latitudes of the Earth’s atmosphere from the vertical temperature profile measured by CHAMP GPS radio occultation on 13 January 2002. The coordinates of sounded region and date of observation are indicated.
Fig. 2. The same as Fig. 1, except from radio occultation conducted in the high latitudes of the Earth's atmosphere on 13 January 2002.
Fig. 3. The same as Fig. 1, except from radio occultation made in the high latitudes of the Earth’s atmosphere on 17 January 2002.
Fig. 4. Example of IGW observations in the middle latitudes of the Earth’s atmosphere from the vertical temperature profile measured by COSMIC GPS radio occultation on 12 June 2009. Description of panels is the same as in Fig. 1.